

# The Extended Helmholtz-Ellis JI Pitch Notation

microtonal accidentals designed by Marc Sabat and Wolfgang von Schweinitz, 2004 (rev. 2018)

## 3-LIMIT (PYTHAGOREAN) INTERVALS

♭♭    ♭    ♮    ♯    ×

## FUNCTION OF THE ACCIDENTALS

notate 35 pitches from the series of untempered perfect fifths  
 $(3/2) \approx \pm 702.0$  cents;  
*perfect fifth (3/2); perfect fourth (4/3); major wholetone (9/8)*

## 5-LIMIT (PTOLEMAIC) INTERVALS

♭♭↓    ♭↓    ♮↓    ♯↓    ×↓    ♭♭↑    ♭↑    ♮↑    ♯↑    ×↑

notate an alteration by one syntonic comma  $(81/80) \approx \pm 21.5$  cents;  
*major third (5/4); minor third (6/5); major sixth (5/3); minor sixth (8/5);  
 minor wholetone (10/9)*

♭♭↓↓    ♭↓↓    ♮↓↓    ♯↓↓    ×↓↓    ♭♭↑↑    ♭↑↑    ♮↑↑    ♯↑↑    ×↑↑

notate an alteration by two syntonic commas  
 $(81/80) \cdot (81/80) \approx \pm 43.0$  cents;  
*augmented fifth (25/16); diminished fourth (32/25)*

♭♭↓↓↓    ♭↓↓↓    ♮↓↓↓    ♯↓↓↓    ×↓↓↓    ♭♭↑↑↑    ♭↑↑↑    ♮↑↑↑    ♯↑↑↑    ×↑↑↑

notate an alteration by three syntonic commas  
 $(81/80) \cdot (81/80) \cdot (81/80) \approx \pm 64.5$  cents;  
*minor diesis (128/125)*

## 7-LIMIT (SEPTIMAL) INTERVALS

♭    ♮

notate an alteration by one septimal comma  $(64/63) \approx \pm 27.3$  cents;  
*natural seventh (7/4); septimal wholetone (8/7);  
 septimal diminished fifth (7/5); septimal tritone (10/7);  
 septimal minor third (7/6); septimal quartertone (36/35)*

♭♭ or ♭♭    ♮♮ or ♮♮

notate an alteration by two septimal commas  
 $(64/63) \cdot (64/63) \approx \pm 54.5$  cents;  
*septimal sixthtone (49/48)*

## 11-LIMIT (UNDECIMAL) INTERVALS

♯    ♮

notate an alteration by one undecimal quartertone  
 $(33/32) \approx \pm 53.3$  cents;  
*undecimal augmented fourth (11/8); undecimal diminished fifth  
 (16/11)*

## 13-LIMIT (TRIDECIMAL) INTERVALS

♮    ♯

notate an alteration by one tridecimal thirdtone  $(27/26) \approx \pm 65.3$   
 cents;  
*tridecimal neutral sixth (13/8); tridecimal neutral third (16/13)*

## PRIMES IN THE HARMONIC SERIES OCTAVE 16 - 32 (5-limit signs are given here relative to "A")

≡♭    ≡♯

notate an alteration of the 5-limit accidental by one 17-limit schisma  
 $(16/17) \cdot (16/15) = (256/255) \approx \pm 6.8$  cents;  
*Galileo's "equal-tempered" semitone (18/17);  
 17-limit diminished seventh chord 10:12:14:17*

~♭    ~♭

notate an alteration by one 19-limit schisma  
 $(19/16) \cdot (27/32) = (513/512) \approx \pm 3.4$  cents;  
*19-limit minor third (19/16); 19-limit minor triad 16:19:24*

↑#

↓b

notate an alteration by one 23-limit comma  
 $(23/16) \cdot (8/9) \cdot (8/9) \cdot (8/9) \approx \pm 16.5$  cents;  
*raised leading tone (23/12)*

≡# or ↑#

≡b or ↓b

notate an alteration of the 5-limit accidental by one 29-limit comma  
 $(29/16) \cdot (5/9) = (145/144) \approx \pm 12.0$  cents

-d

+f

notate an alteration of the 11-limit accidental by one 31-limit schisma  
 $(32/31) \cdot (32/33) = (1024/1023) \approx \pm 1.7$  cents

### PRIMES IN THE HARMONIC SERIES OCTAVE 32 - 64 *(5-limit signs are given here relative to "A")*

{S}f

{S}d

notate an alteration of the 11-limit accidental by one 37-limit schisma  
 $(36/37) \cdot (33/32) = (297/296) \approx \pm 5.8$  cents

{#}

{b}

notate an alteration of the 5-limit accidental by one 41-limit schisma  
 $(32/41) \cdot (81/64) \cdot (81/80) = (6561/6560) \approx \pm 0.3$  cents

{↑}

{↓}

notate an alteration by one 43-limit comma  
 $(43/32) \cdot (3/4) = (129/128) \approx \pm 13.5$  cents

{F}# or {rr}#

{b}b or {ll}b

notate an alteration of the 7-limit accidental by one 47-limit schisma  
 $(32/47) \cdot (48/49) \cdot (3/2) = (2304/2303) \approx \pm 0.8$  cents

{≡}#

{≡}b

notate an alteration of the 5-limit accidental by one 53-limit comma  
 $(32/53) \cdot (5/3) = (160/159) \approx \pm 10.9$  cents

{#}

{d}

notate an alteration of the 13-limit accidental by one 59-limit schisma  
 $(32/59) \cdot (24/13) = (768/767) \approx \pm 2.3$  cents

{r}#

{l}b

notate an alteration of the 7-limit accidental by one 61-limit schisma  
 $(61/32) \cdot (21/40) = (1281/1280) \approx \pm 1.4$  cents

### IRRATIONAL AND TEMPERED INTERVALS

bb

b

q

#

x

notate the respective Equal Tempered Semitone;  
may be combined with a cents indication to notate any pitch

### NOTE ABOUT CENTS INDICATIONS

optional cents indications may be placed above or below the respective accidentals and are always understood in reference to Equal Tempered semitones, as implied by the Pythagorean accidentals if the cents exceed  $\pm 50$  the closest pitch as indicated on a tuner may be written as text, e.g. F#-35

### TEXT NOTATION

in addition to the accidentals, a useful text shorthand for musicians combines the prime constituents of a ratio with the symbols  $\cup$  and  $^\circ$  to indicate harmonic space coordinates: for example  $7^\circ$  or  $\cup 11$

### FONT

The HEJISMuFL font used here (2018) is freely available for download from [www.plainsound.org](http://www.plainsound.org)

\* special thanks to Juhani Nuorvala for suggesting use of a distinct alternate symbol for 29°