

# The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020 | LEGEND

revised by Marc Sabat and Thomas Nicholson | PLAINSOUND MUSIC EDITION | [www.plainsound.org](http://www.plainsound.org)

in collaboration with Wolfgang von Schweinitz, Catherine Lamb, and M.O. Abbott, building upon the original HEJI notation devised by Marc Sabat and Wolfgang von Schweinitz in the early 2000s

## PYTHAGOREAN JUST INTONATION | generated by multiplying / dividing an arbitrary reference frequency by PRIMES 2 and 3 only

...  $\flat\flat$   $\flat$   $\natural$   $\sharp$   $\times$  ...  
note a series of **perfect fifths** above / below a reference  
 $3/2 \approx \pm 702.0$  cents (i.e. 2c wider than tempered)  
each new accidental represents 7 fifths, altering by one apotome  
 $2^{187}/3^{2048} \approx \pm 113.7$  cents

Frequency ratios including higher prime numbers (5–47) may be notated by adding the following distinct accidental symbols. Custom indications for higher primes or various enharmonic substitutions may be invented as needed by simply defining further symbols representing the relevant ratio alterations.

## PTOLEMAIC JUST INTONATION | PRIMES up to 5

$\flat\flat$   $\flat$   $\natural$   $\sharp$   $\times$   $\flat\flat$   $\flat$   $\natural$   $\sharp$   $\times$   
includes the consonant **just major third**  
 $5/4 \approx \pm 386.3$  cents (ca. 14c narrower than tempered)  
alteration by one syntonic comma  
 $81/80 \approx \pm 21.5$  cents  
 $\flat\flat$   $\flat$   $\natural$   $\sharp$   $\times$   
alteration by two syntonic commas  
 $81/80 \cdot 81/80 \approx \pm 43.0$  cents  
 $\sim\sharp = \flat$   $\sim\flat = \sharp$   
alteration by one schisma to notate an exact enharmonic substitution  
 $32805/32768 \approx \pm 2.0$  cents

## SEPTIMAL JI | PRIME 7

$\flat$   $\natural$   
includes the consonant **natural seventh**  
 $7/4 \approx \pm 968.8$  cents (ca. 31c narrower than tempered)  
alteration by one septimal comma (Giuseppe Tartini)  
 $64/63 \approx \pm 27.3$  cents  
 $\flat$   $\natural$   
alteration by two septimal commas  
 $64/63 \cdot 64/63 \approx \pm 54.5$  cents

## UNDECIMAL | PRIME 11

$\flat$   $\natural$   
includes the **undecimal semi-augmented fourth**  
 $11/8 \approx \pm 551.3$  cents (ca. 51c wider than tempered)  
alteration by one undecimal quartertone (Richard H. Stein)  
 $33/32 \approx \pm 53.3$  cents

## TRIDECIMAL | PRIME 13

$\flat$   $\natural$   
includes the **tridecimal neutral sixth**  
 $13/8 \approx \pm 840.5$  cents (ca. 59c narrower than a tempered major sixth)  
alteration by one tridecimal thirddtone (Gérard Grisey)  
 $27/26 \approx \pm 65.3$  cents

## PRIMES 17 THROUGH 47

$\flat$   $\natural$   
alteration by one 17-limit schisma  
 $2^{187}/3^{2176} \approx \pm 8.7$  cents  
 $\flat$   $\natural$   
alteration by one 19-limit schisma  
 $5^{13}/7^{512} \approx \pm 3.4$  cents  
 $\flat$   $\natural$   
alteration by one 23-limit comma (James Tenney / John Cage)  
 $7^{36}/2^{729} \approx \pm 16.5$  cents  
 $\flat$   $\natural$   
alteration by one 29-limit sixthtone  
 $2^{61}/3^{256} \approx \pm 33.5$  cents  
 $\flat$   $\natural$   
alteration by one 31-limit quartertone (Alinaghi Vaziri)  
 $3^{32}/2^{31} \approx \pm 55.0$  cents  
 $\flat$   $\natural$   
alteration by one 37-limit quartertone (Ivan Wyschnegradsky)  
 $3^{37}/2^{36} \approx \pm 47.4$  cents  
 $\flat$   $\natural$   
alteration by one 41-limit comma (Ben Johnston)  
 $8^{82}/3^{81} \approx \pm 21.2$  cents  
 $\flat$   $\natural$   
alteration by one 43-limit comma  
 $1^{129}/2^{128} \approx \pm 13.5$  cents  
 $\flat$   $\natural$   
alteration by one 47-limit quartertone  
 $7^{52}/2^{729} \approx \pm 53.8$  cents

**CENTS** HEJI accidentals may be combined with an indication of their deviation in cents from equal temperament as read on a tuning meter; A $\natural$  440 Hz is usually defined to be  $\pm 0$  cents. If this deviation exceeds  $\pm 50$  cents, the nearest tempered pitch-class may be added: e.g. A $\flat$  (–65 cents from A $\natural$ ) could include the annotation A $\flat$ +35 placed above or below its accidental.

## TEMPERED NOTES | may be combined with cents deviations to notate free microtonal pitches

...  $\flat\flat$   $\flat$   $\natural$   $\sharp$   $\times$  ...  
indicate the respective equal tempered quartertone;  
show which pitch is assigned a deviation of 0c