

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020 | LEGEND

revised by Marc Sabat and Thomas Nicholson | PLAINSOUND MUSIC EDITION | www.plainsound.org

in collaboration with Wolfgang von Schweinitz, Catherine Lamb, and M.O. Abbott, building upon the original HEJI notation devised by Marc Sabat and Wolfgang von Schweinitz in the early 2000s

PYTHAGOREAN JUST INTONATION | generated by multiplying / dividing an arbitrary reference frequency by PRIMES 2 and 3 only

... ♭♭ ♭ ♮ # ✖ ...

notate a series of **perfect fifths** above / below a reference — $3/2 \approx \pm 702.0$ cents (ca. 2c wider than tempered)
each new accidental therefore represents 7 fifths, altering by one apotome — $2^{187}/2048 \approx \pm 113.7$ cents

Frequency ratios including higher prime numbers (5–47) may be precisely written by adding the following distinct accidental symbols. Custom indications for higher primes or various enharmonic substitutions may be invented as needed by simply defining further symbols representing the relevant ratio alterations.

PTOLEMAIC JUST INTONATION | PRIMES up to 5

♭♭↓ ♭↓ ♮↓ #↓ ✖↓
♭♭↓ ♭↓ ♮↓ #↓ ✖↓
~# = ♭

♭♭↑ ♭↑ ♮↑ #↑ ✖↑
♭♭↑ ♭↑ ♮↑ #↑ ✖↑
~♭ = #

includes the consonant **just major third** — $5/4 \approx \pm 386.3$ cents (ca. 14c narrower than tempered)

alteration by one syntonic comma — $81/80 \approx \pm 21.5$ cents

alteration by two syntonic commas — $81/80 \cdot 81/80 \approx \pm 43.0$ cents

alteration by one schisma to notate an exact enharmonic substitution — $3^{2805}/32768 \approx \pm 2.0$ cents

SEPTIMAL JI | PRIME 7

↳
↯

↱
↷

includes the consonant **natural seventh** — $7/4 \approx \pm 968.8$ cents (ca. 31c narrower than tempered)

alteration by one septimal comma — $64/63 \approx \pm 27.3$ cents (Giuseppe Tartini)

alteration by two septimal commas — $64/63 \cdot 64/63 \approx \pm 54.5$ cents

UNDECIMAL | PRIME 11

↻

↻

includes the **undecimal semi-augmented fourth** — $11/8 \approx \pm 551.3$ cents (ca. 51c wider than tempered)

alteration by one undecimal quartertone — $33/32 \approx \pm 53.3$ cents (Richard H. Stein)

TRIDECIMAL | PRIME 13

↻

↻

includes the **tridecimal neutral sixth** — $13/8 \approx \pm 840.5$ cents (ca. 59c narrower than a tempered major sixth)

alteration by one tridecimal thirdtone — $27/26 \approx \pm 65.3$ cents (Gérard Grisey)

HIGHER PRIMES 17 – 47

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≍

alteration by one 17-limit schisma — $2^{187}/2176 \approx \pm 8.7$ cents

↘

↙

alteration by one 19-limit schisma — $5^{13}/512 \approx \pm 3.4$ cents

↓

↑

alteration by one 23-limit comma — $7^{36}/729 \approx \pm 16.5$ cents (James Tenney / John Cage)

↘

↑

alteration by one 29-limit sixthtone — $2^{61}/256 \approx \pm 33.5$ cents

↘

↘

alteration by one 31-limit quartertone — $3^2/31 \approx \pm 55.0$ cents (Alinaghi Vaziri)

↘

↘

alteration by one 37-limit quartertone — $3^7/36 \approx \pm 47.4$ cents (Ivan Wyschnegradsky)

−

+

alteration by one 41-limit comma — $8^2/81 \approx \pm 21.2$ cents (Ben Johnston)

↘

↗

alteration by one 43-limit comma — $1^{29}/128 \approx \pm 13.5$ cents

↘

↘

alteration by one 47-limit quartertone — $7^{52}/729 \approx \pm 53.8$ cents

CENTS HEJI accidentals may be combined with an indication of their deviation in cents from equal temperament as read on a tuning meter; A♮ 440 Hz is usually defined to be ± 0 cents. If this deviation exceeds ± 50 cents, the nearest tempered pitch-class may be added: e.g. A[♯] (−65 cents from A♮) could include the annotation A♯+35 placed above or below its accidental.

TEMPERED NOTES | may be combined with cents deviations to notate free microtonal pitches

... ♭♭ ♭ ↻ ♮ ↘ # ✖ ...

indicate the respective equal tempered quartertone; show which pitch is assigned a deviation of 0c

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

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Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard utonal notation above ♭A

8₁ partial interval alteration

+2 -14 -31 -14 +4 +51 +2 +5 +4 +51 +28 +2

5° M3 (81:80)

7° m7 (64:63)

11° P4 (32:33)

13° M6 (27:26)

17° aug8 (2187:2176)

19° m3 (512:513)

23° aug4 (729:736)

F+41 -31 -12 F+41 -31 -12 -29 -10

29° m7 (256:261)

31° P8 (32:31)

37° M2 (36:37)

41° M3 (81:82)

43° P4 (128:129)

47° aug4 (729:752)

+6 +30 +5 +4 +51 +29 +12 +51 +28 E-34 +2 E+38

Standard utonal notation below ♭E

+2 8₁ +2 +2 +16 +33 +2 +16 G#-39 +33 +14 +2 +4 +16 +31

u5 M3 (80:81)

u7 m7 (63:64)

u11 P4 (33:32)

u13 M6 (26:27)

u17 aug8 (2176:2187)

u19 m3 (513:512)

u23 aug4 (736:729)

+29 G#-39 +33 +14 F-43 +2 +47 +4 C#-41 +16 +31 +12

u29 m7 (261:256)

u31 P8 (31:32)

u37 M2 (37:36)

u41 M3 (82:81)

u43 P4 (129:128)

u47 aug4 (752:729)

-4 -28 -51 -3 -2 -49 -27 -10 -49 -26 A+36 A-36

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PYTHAGOREAN JUST INTONATION | generated by multiplying / dividing an arbitrary reference frequency by PRIMES 2 and 3 only

... $\flat\flat$ \flat \natural \sharp \times ...

notate a series of **perfect fifths** above / below a reference
 $3/2 \approx \pm 702.0$ cents (i.e. 2c wider than tempered)
 each new accidental represents 7 fifths, altering by one apotome
 $2187/2048 \approx \pm 113.7$ cents

Frequency ratios including higher prime numbers (5–47) may be notated by adding the following distinct accidental symbols. Custom indications for higher primes or various enharmonic substitutions may be invented as needed by simply defining further symbols representing the relevant ratio alterations.

PTOLEMAIC JUST INTONATION | PRIMES up to 5

$\flat\flat$ \flat \natural \sharp \times $\flat\flat$ \flat \natural \sharp \times

includes the consonant **just major third**
 $5/4 \approx \pm 386.3$ cents (ca. 14c narrower than tempered)

alteration by one syntonic comma
 $81/80 \approx \pm 21.5$ cents

$\flat\flat$ \flat \natural \sharp \times $\flat\flat$ \flat \natural \sharp \times

alteration by two syntonic commas
 $81/80 \cdot 81/80 \approx \pm 43.0$ cents

$\sim\sharp = \flat$ $\rightarrow\flat = \sharp$

alteration by one schisma to notate an exact enharmonic substitution
 $32805/32768 \approx \pm 2.0$ cents

SEPTIMAL JI | PRIME 7

\flat \natural

includes the consonant **natural seventh**
 $7/4 \approx \pm 968.8$ cents (ca. 31c narrower than tempered)

alteration by one septimal comma (Giuseppe Tartini)
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includes the **undecimal semi-augmented fourth**
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alteration by one undecimal quartertone (Richard H. Stein)
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TRIDECIMAL | PRIME 13

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alteration by one tridecimal thirddtone (Gérard Grisey)
 $27/26 \approx \pm 65.3$ cents

PRIMES 17 THROUGH 47

\flat \natural

alteration by one 17-limit schisma
 $2187/2176 \approx \pm 8.7$ cents

\flat \natural

alteration by one 19-limit schisma
 $513/512 \approx \pm 3.4$ cents

\flat \natural

alteration by one 23-limit comma (James Tenney / John Cage)
 $736/729 \approx \pm 16.5$ cents

\flat \natural

alteration by one 29-limit sixthtone
 $261/256 \approx \pm 33.5$ cents

\flat \natural

alteration by one 31-limit quartertone (Alinaghi Vaziri)
 $32/31 \approx \pm 55.0$ cents

\flat \natural

alteration by one 37-limit quartertone (Ivan Wyschnegradsky)
 $37/36 \approx \pm 47.4$ cents

\flat \natural

alteration by one 41-limit comma (Ben Johnston)
 $82/81 \approx \pm 21.2$ cents

\flat \natural

alteration by one 43-limit comma
 $129/128 \approx \pm 13.5$ cents

\flat \natural

alteration by one 47-limit quartertone
 $752/729 \approx \pm 53.8$ cents

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indicate the respective equal tempered quartertone;
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Standard utonal notation above $\flat A$

Standard utonal notation above $\flat A$ musical notation showing intervals and ratios:

- partial interval alteration: 8^{\perp}
- $+2$ interval: M3 (81:80)
- $+2$ interval: m7 (64:63)
- $+4$ interval: P4 (32:33)
- $+51$ interval: M6 (27:26)
- $+2$ interval: P8 (32:31)
- $+5$ interval: aug8 (2187:2176)
- $+4$ interval: m3 (512:513)
- $+51$ interval: aug4 (729:736)
- $+28$ interval: m7 (256:261)
- $+2$ interval: P8 (32:31)
- $+6$ interval: P4 (128:129)
- $+30$ interval: aug4 (729:752)
- $+5$ interval: M2 (36:37)
- $+4$ interval: M3 (81:82)
- $+51$ interval: P4 (128:129)
- $+29$ interval: M3 (81:82)
- $+12$ interval: P4 (128:129)
- $+51$ interval: P4 (128:129)
- $+28$ interval: aug4 (729:752)
- $E-34$ interval: aug4 (729:752)
- $+2$ interval: P8 (32:31)
- $E+38$ interval: P8 (32:31)

Standard utonal notation below $\flat E$

Standard utonal notation below $\flat E$ musical notation showing intervals and ratios:

- $+2$ interval: M3 (80:81)
- $+2$ interval: m7 (63:64)
- $+16$ interval: P4 (33:32)
- $+33$ interval: M6 (26:27)
- $+2$ interval: P4 (33:32)
- $+16$ interval: P4 (33:32)
- $+33$ interval: M6 (26:27)
- $+14$ interval: P8 (31:32)
- $+2$ interval: aug8 (2176:2187)
- $+4$ interval: m3 (513:512)
- $+16$ interval: aug4 (736:729)
- $+31$ interval: m7 (261:256)
- $+29$ interval: P8 (31:32)
- $G\#-39$ interval: P8 (31:32)
- $+33$ interval: P8 (31:32)
- $+14$ interval: P8 (31:32)
- $F-43$ interval: P8 (31:32)
- $+2$ interval: M2 (37:36)
- $+47$ interval: M3 (82:81)
- $+4$ interval: P4 (129:128)
- $C\#-41$ interval: P4 (129:128)
- $+16$ interval: P4 (129:128)
- $+31$ interval: P4 (129:128)
- $+12$ interval: aug4 (752:729)