Introducing my work in microtonally extended just intonation

by Marc Sabat – edited from a lecture presented at the 2017 Ostrava Days

I'm really glad to be here, my name is Marc Sabat: I'm a composer with a background as a performer. I studied the violin for many years – some of you might know recordings I made with pianist Stephen Clarke of Morton Feldman's music[1], or music of James Tenney[2], for example – at the same time, since 1992 or so I've been active professionally as a composer. I'm inspired by learning about the possibilities of what I'd call a kind of microtonal just intonation, or *microtonally extended just intonation*. That's something that I want to introduce to you briefly in my talk this morning.

I'm sure all of you are familiar with the term "just intonation" (or "JI" for short). It comes up often in music theory, or when talking about various musical cultures that employ it – let's say, as a kind of underpinning, – which in the last four or five hundred years we have not. There have always been occasional theorists, mathematicians, and some composers – but mostly coming from the theory and sound/acoustics side – who have said: "wouldn't it be interesting to try to consider just intonation?" But the composers who have taken it up only start to emerge around 1880. I don't know if any of you are connected to me on Facebook, you may have seen a post I did a couple weeks back about just intonation composers. I made a list off the top of my head and posted it online.[3] Lots of people have responded and added more names to it, now it is a work-in-progress and you start to see how many different people have been exploring this idea in their work over the past 150 years.

Around the time of the Renaissance, there were a couple of theorists who did experimental compositions in JI. In the 1500's Giambattista Doni wrote contrapuntal music using old Ptolemaic modes on violins and viola da gambas. He built a whole family of instruments for performing in his just intonation tone system. He wrote several examples, very odd little pieces that have yet to be played and recorded – that's something that ought to be done. These are the first European pieces I know of based on 11-limit harmonic ratios. And then there's another Giambattista. Giambattista Benedetti is the inventor of what James Tenney later called harmonic distance, namely, a measure of harmonicity – a very simple one, not as mathematically involved as ones introduced by, for example, Leonhard Euler or Clarence Barlow. But the Benedetti-Tenney distance is actually quite a good predictor of relative consonance, namely of how difficult it is for us to perceive the specific character of an interval.[4] There is potentially an *infinite* number of possible intervals expressed as frequency ratios and a flexible but *finite* subset of these that we can perceive: that's something that underlies the work I've been doing, getting to know the tuneable intervals and composing music with them.

So just intonation as a composition project is something which kind of wakes up in the late 19th century after proposals made by Helmholtz in his important book with the fantastic title "On the Sensations of Tone as a Physiological Basis for the Theory of Music". All of you probably know the studies he made of

1 Release on mode records, Morton Feldman, *Complete music for violin and piano* (2000).
2 Release on hat[now]ART, James Tenney, *Music for violin and piano* (1999).
3 https://docs.google.com/spreadsheets/d/1anzjL32a2D79nVofBCxZgDoyE1RmnT_Ct1kXyohe6-A/edit?usp=sharing
4 For a given interval, expressed as a ratio *b/a* in lowest terms, its Benedetti distance is the product *ab*. In musical terms, 1 represents the periodicity pitch and *ab* the least common partial (the first frequency common to the harmonic series of both pitches). Roughly speaking, if the periodicity pitch is >25 Hz and the common partial is <5000 Hz, i.e. if both frequencies fall approximately within the range of the modern piano, there is a good chance that the ratio will be a tuneable interval. In other words, the Benedetti distance of a tuneable interval will not exceed – for the most part – 200. Tenney's *harmonic distance* measure is $\log_2(ab)$, which then correlates with the common measurement used to express interval size or *melodic distance*, called *cents* and evaluated by calculating $1200\log_2(b/a)$, effectively dividing the octave into 1200 equal parts.

the physiology of the ear and how it relates to the way we perceive sound, working as a filter and breaking down sound into different frequencies, which are then recombined within the brain. The physical experience of sound, and our mental efforts to reconstruct it, are fundamentally connected to the idea of harmony. The first composer that I have on my list currently, who actually took up the idea of writing pieces in just intonation, is a very little known Australian composer, Elsie Hamilton, who was connected to Kathleen Schlesinger. Schlesinger did studies of the Greek aulos, which many musicologists think are not historically valid in terms of what they reveal about Ancient Greek music, but it's certainly a very interesting creative undertaking. She set up a system of subharmonic scales based on equal division holes on flutes, and Elsie Hamilton composed pieces based on these ratios. The American JI pioneer Harry Partch also met Schlesinger and was impressed by her ideas.

So, what does it mean to approach just intonation not just theoretically but compositionally? It's a very simple matter. It's really about conceiving of the relationships of tones – of frequencies, which could be harmonic fundamentals or electronic tones, or even any complex sounds with salient frequencies – conceiving them in terms of specific proportions, of intervals. Now, these proportions could perhaps even take an irrational form, they don't necessarily have to be proportions of whole numbers. So, I would say that way one could even embrace the kinds of tone systems that we call irrational tone systems: "tempered" systems, or various "equal division" systems, like in Wyschnegradsky's work. But my feeling is that which is most *unfamilia*r to us, what sparked my own interest in this topic, is to think about how musical intervals, their consonance and dissonance, may be most usefully understood in terms of rational numbers. So, what does that really mean? Essentially, it means that there is something underlying harmonic experience which is *not* about the acoustic spectrum, it's not a *spectral* idea, it's not an idea about timbre, it's actually an idea about *the harmonic series as an abstract objec*t, as a *model* of *the only chord there is.*

The harmonic series is a mathematical construct, a series of integer proportions, which sets up a way of considering, of measuring or modelling frequency relationships. Frequencies may be grouped to form possible subsets of harmonic series, sometimes with fundamentals going even down to the sub-audio range. These produce structures which can be thought of as specific, distinct entities – specific chords, specific intervals, specific modes if you like, specific tonal subsets with distinct soundings, distinct *characters*. You can see this most clearly from the point of view of intervals – I'll show you in a little bit on the violin how I do this – but it is something which can also be applied to chords and larger sets of pitches. The tonal framework that most of us are accustomed to is something like, for example, the chromatic scale in 12-tone equal temperament.[5] Rather than taking this "artificial" system as the central backbone upon which various proportions can then be constructed, I tend to approach it the other way around. Namely, I think we hear the tempered tones, or any other notes that are given in any system, *in relation to each other based on just intonation,* on the principles of the harmonic series. That's how we can understand them as a relationship, interval, chord, melody, etc. Now, that may be a slightly unusual way to look at it, but it reflects my own experience, which is that the intonation of intervals or sounds, the production of tonal relationships on an instrument where there are no fixed keys, no pre-tuned elements, is something which you have to decide on by ear. You have to hear it, you have to be able to

5 The general style of describing equal division systems is to state the number of divisions follwed by ED and then the interval being divided. In most cases, this is the octave, which may be be written as the ratio 2/1 or simply 2. Thus, the conventional 12-tone equal temperament can be written, more precisely, as 12-ED2. Various other equal division systems more exactly represent some of the harmonic intervals, but at the expense of having more tones in one octave. Thus 72-ED2 is often proposed for approximating 11-limit harmonic spaces, while retaining the conventional scale as a subset (composers who have used it in some works include James Tenney, Ezra Sims, Joe Maneri, Hans Zender, Georg Friedrich Haas, Marc Sabat). Other significant systems include 31-ED2 (extended meantone) and 53-ED2 (closely approximating 5-limit JI).

hear a specific relationship, to be able to tune it. I suppose this grew out my experience playing the violin.

Now, you could start by hearing a categorical relationship, which carries a certain degree of meaning – for example this [plays an interval on violin] – that's a slightly small, equal tempered fifth, two cents *narrower* than the just intonation fifth. It's not an interval which is tuned according to a whole number proportion of smaller numbers.[6] As it stands, it would require a proportion of very large whole numbers to approximate it, but our ear is not able to *recognise* its sound as a distinct, simple relationship of numbers. So, we can't tune it directly without invoking some kind of outside measurement, for example by counting beats or using an electronic tuner. Instead, the equal tempered fifth sounds as it does on the piano, a little bit "beating" or "out of tune", but in such a subtle manner that the ear easily accepts it as though it were in tune. On the other hand, if we're tuning up a violin [tunes violin by playing and adjusting fifths], I begin by producing a fifth approximately, and then alter one of the two notes to precisely tune the interval as a simple whole number proportion, 2:3, by listening to how the combination sounds. If I verify the result, you'll find that each fifth is tuned slightly larger than tempered, by about 2 cents. This is a tiny amount, but it's the way we actually hear the interval, so there's absolutely no difficulty making it accurate.

Whenever I produce two sounds simultaneously, I'm basically producing an *interaction* of tone. No matter how they're tuned, I'm producing and hearing a composite sound, which is caused by sensing interactions of their largely harmonic partials. If you want to be really specific, you can deal with this on a case-by-case basis, sounds-specific, which is like the approach found in spectral music. That's something that interests me less, what interests me more is the idea that by thinking in terms of proportions, in terms of identifiable, tuneable sounds, it's actually possible to come up with something which is like a more general approach to harmony, something which is transferable from one specific timbral situation to another. You know, the nature of harmonic perception is connected to our ability to perceive something like a voice, or any sound which is the consequence of a single action, a single source of vibration. So, we have all kinds of auditory schemes for unravelling that kind of complex information.

I'd like to start now by playing part of the piece of music which will get performed here in a week, a live recording from the premiere, just so you can get a feel for this approach, in the context of a piece of music that I have written recently, as I've been writing in the last, let's say, four or five years. Then I'll talk a little about how I developed up to that point, and show some earlier pieces which deal with some of these ideas, for example, in a much more specific or reduced way.

So in this interpretation, the piece, called "Lying in the grass, river and clouds", is in total about 35 minutes long, and we'll listen to maybe the first 10 minutes or so. It was written in 2012. To give you a little bit of context, it's part of a cycle of pieces, which in total may be presented as an entire work, as a kind of installation, all deriving from a common principle that I unfolded in different ways, and on different time-scales. This is the fourth part and it is generally presented as a concert piece.

[piece plays]

6 The exact irrational proportion of a fifth in 12-ED2 is expressed as $2^{7/12}/1$.

So that was an ensemble of 15 instruments, with a central part played on the piano. The piano is tuned, when possible, in a completely unstretched, exact equal temperament, but as you can hear, the piano, though it's continually active, is composed so its pitches are always presented in relation to the harmonies created by the other instruments, rather than the other way around. The instruments don't adjust or orient themselves to the tuning of the piano. My feeling about the equal tempered piano, the way I am writing for it here, grew out of a sense that its very nature is a non-harmonic nature. Although equal temperament can be used to approximate harmonic events, it doesn't really represent them, doesn't represent their *sound*. It *suggests* some relationships or ratios *without actually activating their sonority*.

My approach, in this particular cycle, was to deal with all of the dyads on the piano as structures which create or evoke a summation and a difference tone. So, thinking about the interactions of each tempered interval exactly as given, as the interaction of two fundamental frequencies, ignoring their lack of common periodicity or their beating, treating those phenomena as colors, and focussing on the *combination tone frequencies*. Then I mapped those resulting microtonal pitches, treating them as material that I could consider as being interconnected with the instruments, which are actually relating to each other as much as possible by a network of whole number ratios. The instruments move harmonically by tuneable intervals, so their fundamentals go very far away, and then return back again, in relation to the tempered frame of reference. They move microtonally within the entire space, accepting the possibility of playing all of the frequencies available between each semitone, and the piano is composed in such a way to create a connection to the harmonies present in the ensemble, either by harmonic or subharmonic, tuneable interval, summation or difference tone, or, when its pitch is actually very close to being part of the structure, directly, expressed within a tolerance of about seven cents. So, using a fine microtonal resolution much smaller than the step size in equal temperament.

I've talked a little about just intonation, meaning the use of ratios and conceiving relationships of frequency as ratios, and as a consequence, *allowing the resulting microtonality to be heard*, to be sounded. The idea of composing with respect to just intonation is not a novelty choice like "OK well, what else could I do?" And it's not an ideological choice. It is simply, for me, motivated by the fact that I really want to hear those sounds. Between a number of composers working in this field, as far as I can tell, there is a wonderful body of *new* music being created, and here I feel like this old catch-phrase actually means something, a varied and diverse repertoire is starting to take shape. Whether you think of the work of James Tenney, or La Monte Young, or Harry Partch, Ben Johnston, Wolfgang von Schweinitz or Catherine Lamb, the list goes on – any number of composers, lots of people of all generations including many extremely talented young composers in their 20's and 30's are exploring what these sounds sound like and what it means *to play* them.

Now, what is *extended* just intonation? What does *microtonal* mean? In the Renaissance time, – I don't know how many of you have read Gioseffo Zarlino? When was it, the 1560s? So, Zarlino writes a lot of things, some of which in retrospect aren't great, in my opinion, like the troubles he had with radical experimental music exploring the chromatic and enharmonic modes – but one thing that he did really get right was when he talks about the various melodic modes spanning different intervals. And he had the point of view, quite interestingly, that when singers sing counterpoint, that they naturally sing simple ratios in just intonation. Vincenzo Galilei thought he was wrong, that singers were far too out-of-tune to manage it, but between the two of them there was still quite a debate about the way the ratios are employed. What's interesting in Zarlino, to me, though, is how he talks about the melodic mode of the major third. The idea is: there are two modes of the major third, sung diatonically, which are whole step, whole step, and then whole step, whole step. Well then, what does that mean? It's the major whole step and the minor whole step. Zarlino distinguished them as being different from each other, and therefore also different from the keyboard tuning, which generally has only one size of whole step.

So how does this come about, and what is that difference between the two whole steps, which became eradicated in the meantone temperament? The point is that we reduced our thinking to one size of tone, "the" wholetone, which was easier to handle though it remained clear to musicians for a long time that there were two kinds of semitones, diatonic and chromatic, expressed by enharmonic spelling. It remained like that through Mozart into Brahms, and then changed in the 20th century as musicians adopted a concept of fully chromatic equal temperament with so-called substitution of enharmonic equivalents. The consequences of this adoption were that simple harmonic relations were for a while reduced or eliminated, and now have become kind of a strange residua, at least in terms of contemporary or experimental music practice. So, the two types of wholetone, how does that come about? What is a tone in this context anyway? It's a harmonic interval fundamentally, but what's a tone? Anyone know?

A: It's a vibration at a particular rhythm.

As an harmonic interval, but when I play a tone [plays], it's a very strong interaction, and as such, as a harmonic event, it's not something like a fifth or a fourth, it's not like a large interval. Why? Why can I tune fifths better than I can tune minor seconds? Why is it more difficult to establish them? It's nothing about the frequencies themselves, it's about our *perception* of the frequencies, and about what aspect dominates. The problem is they're vibrating too close to each other inside the ear, so their interaction is stronger than the two individual vibrating frequencies. The sounds themselves are compressed within a critical band, and therefore they create an interference that dominates our perception. It's much more difficult – not impossible, but much more difficult – to actually establish the interval. And that line happens approximately between the minor third and the wholetone. It's even wider in the bass register.

So, the tone and all kinds of *micro*-tones are not perceived in the same way, they do not produce the same kind of cognitive phenomenon as a larger interval. The interval is an harmonic event, a simultaneous sounding, directly verifiable. A tone or smaller may primarily be established as the *difference* of two harmonic events, of two larger intervals, of two *consonances*. That is my approach to it, and that's also the way it's often defined in early texts, for example you find it in Rameau's books about harmony. This conception leads me to explore composition of just intonation counterpoints, based on tuneable interval sounds or periodic signatures, as I call them. But if the smaller, let's call them *melodic* intervals are differences between consonances, it clearly depends on who's defining it.[7] Some writers have made as many as seven categories of consonance, some have just the simplest ones: fifths and fourths and octaves. Some call the fourth dissonant, because the upper tone isn't in the harmonic series of the lower one.

But basically, to return to my question: a tone is a *difference*, and it's a difference between what intervals? First of all, it's the difference between a fifth and a fourth. The difference between a perfect fifth and a perfect fourth is a tone, and a tone in this case is the relationship of eight to nine. In the sense that the perfect fifth is defined by the three to two relationship, and the perfect fourth by the four to three

7 James Tenney, in his survey *A History of Consonance and Dissonance* (1988), discusses the emergence of various "consonance/dissonance concepts" in a historical context drawing on various theoretical and musical practices.

relationship. So, how do we find the difference between them? What's the actual process if you have 3/2, and you have 4/3, what does that actually mean? If we also know that the octave is 2/1, what does that all actually say?

What's most fun, I think, is experiencing the *sound* of these things. Not experiencing them only as a sensory novelty effect, like in pieces of music where someone throws in the harmonic series, and you can hear all of the early harmonics and how they blend, or fuse. The exciting thing, for me, is what various subsets sound like, their individual qualities and colors, harsh, gentle, muddy, bright, and the infinite contexts they create beside each other – *that's* what's musically interesting. Because subsets of the harmonic series sound really different from each other. And they come into being really naturally. So, let's start a harmonic series, and let's say just for the hell of it that 1 is in this case the note C. And we have this sort of situation: the octave of everything is double, so the octave of 1 is 2, of 2 is 4, of 4 is 8, all called C. The octave of 3 is 6, both are G, 8 is C, and 9 is D, a tone higher. So here again is one, two, three of another kind. Just depends on what the fundamental is to determine the numbers, the relationships.

If I do this on the violin, transposed to my open strings, here I'm playing the interval 3/2 [plays A and D], and then I'm playing 4/3 [plays A and E]. It's a big tone, slightly bigger than on this piano, of 204 cents. So here it is, 8:9, in this case from D up to E, which equals 204 cents, played exacly as the difference of two consonances. That's a *major* wholetone.

Going back, now: we have our C series – now let's take the E, in this context, in relation to the G, what harmonic is relevant here? It's the one that's in between 4 and 6. It completes the harmonic series major chord. What is the name of this interval then, as a ratio? – 5/3. So now [plays fifth followed by sixth] I'm moving from 3/2 to 5/3. And this new E, compared to the previous one, is lower by a *syntonic comma*, by an interval which is perhaps only rarely treated as melodic [demonstrates two different E's], because strictly speaking it is not a difference of consonances but in fact once removed, a *difference of differences*![8] I've written a piece where I think it's melodic, but that's a matter of opinion. But generally, it's not so melodic. It was something which was studied in great detail in the renaissance. Francisco de Salinas had a special instrument built where one manual was in just intonation, all by ratio, and where the other manual was in the *Sistema Partecipato*, what we now call Meantone, which means any system which has only one size wholetone. Now what is the classical Sistema Partecipato? In a nutshell, it is basically fucking up D so as to have a good major third. So if you look at this right here, C, D and E. What is the melody C D E in the harmonic series? C to D is a fourth to a fifth, so it's a major tone. 8:9. The major third from C to E is going from 4 to 5, so it's the same an octave higher, we call it 8:10. So, C to D to E is 8:9:10. You have two different proportions for the two wholetones (8:9 and 9:10). These are proportions of successive harmonics, which are often the most melodically viable intervals because they are neighboring harmonic structures. We call 9:10 a *minor* whole tone, and the difference between the major and minor tone is 81/80, called syntonic comma. Meantone splits the comma in two and evens out the tones.

8 Composer Thomas Nicholson distinguishes *enharmonics* (small intervals with different spellings, like G-sharp and A-flat) from *commas* (microtonal variants of a single note, like E-Pythagorean and E-syntonic-comma-down). Classical notation is able to express enharmonic differences (augmented and diminished intervals), but historically avoided explicit notation of commas, which are necessary to differentiate harmonic ratios based on different primes. Triadic music, tuned in JI, is based on the primes 2, 3 and 5, but only 2 and 3 are represented in conventional staff notation. The Extended Helmholtz-Ellis JI Pitch Notation, developed by Marc Sabat and Wolfgang von Schweinitz, extends the staff notation with various commas associated with higher primes (5,7,11,13, etc.) to permit exact notation of both enharmonic and comma differences.

As you work with just intonation, like I did in my big ensemble piece, the network of relationships modulates into all dimensions. This you can see very easily right here in the example we were talking about: if A is common to both intervals, what's the root in each case? A to D is 3/2, so what's the fundamental? D. Now, when I play A and E, the ratio changes to 4/3. A remains the same, but in the ratio A changes its *role* from being the "3" to being the "4", so the fundamental is now A. In the melody D-E played as 8:9, the implied root movement, the movement of fundamentals, is therefore D going to A.

Basically, an *interval*, which is a proportion, is always made of *two elements from some harmonic series*. The *fundamental* that I'm talking about is *the note which is called "1" in that particular series*. So, when you have a perfect fourth making a relationship of 3:4 between two notes, we can easily determine what root fundamental generates these two harmonics, and shares them. The feeling of consonance or harmonicity that we sense is built upon our ability to perceive, as a kind of shadow within the interval sonority, its *periodicity*, the frequency of the virtual fundamental.[9] It's a key element of how harmony works. So, there's a reason why root movements by fifths or thirds are quite natural and appealing to the ear, because they're coming out of the differences of simple consonances, and the differences of simple consonances give us the first melodic intervals.

We've experienced two types of wholetone, and the difference between the two wholetones which I demonstrated here between the two e's, that's what is called the comma. It's a microtone, a small fraction of a tone. What is the ratio of the comma, how do we figure that out? We need to figure out the difference between the E from the series of A, and the E from the series of C. How do we get to it? The way the fifths work is they build up, one on top of the next, each serves as a fundamental generating the next one, since the ratio of the fifth is 2:3 and 2 represents the pitch of the fundamental.

[writes on board: notes C G D A E, ratios connecting them and harmonic series numbers]

The relationships here are *not* meant to be tempered notes – the notation I'm writing here, using the standard accidentals, implies notes *related to each other by perfect fifths*. Remember that the tempered system is just an approximation, and not the real thing in the sense that the real thing is just those relationships that we can build up by simple proportions. Let's imagine we tuned the A of the piano, and then all of these notes that are related by fifths are related by the proportion 3:2 from this A. Not a "circle" but simply a series, or chain, of fifths. That would be the ancient system, the Pythagorean diatonic frame, which is the basis of Indian music, Arabic music, and our music until around 1500. The Pythagorean diatonic frame is the harmonics that are produced only by multiples of 2 and 3, for example, starting from 6 [writes notes on board G C D G C D G A C D]: 6, 8, 9, 12, 16, 18, 24, 27, 32, 36 and so on. The fundamental is now a low C which is the octave below this one [points to pitch].

These are the numbers of the harmonic series – I'm not talking about a spectrum here – I'm taking the real harmonic series as a mathematical concept, as a way of defining the relationships of the frequencies as a tuning, like I talked about earlier. So, you see, as you build up a harmonic series, the reason that 2 and 3 and their multiples are crucial to developing the basis or spine of the tone system is they're the only notes that repeat often in the early numbers of the harmonic series, the only ones that we actually hear forming their own harmonic series, and in that way forming a backbone, structures within the structure. Here, look at it: Fourth, wholetone, fourth, fourth, wholetone, fourth, wholetone, a special kind

9 The harmonicity of an interval, in this sense, has to do with how fully the respective harmonics of the two notes represent the harmonics of the virtual fundamental or periodicity pitch, up to the common partial where the two series meet. James Tenney described this property as the *intersection* of an interval and its implied harmonic series.

of minor third, (27 to 32 is a Pythagorean minor third), whole tone. You have a series of fifths, 8 to 12 is C to G; 6 to 9, which is also 12 to 18 (same pitches an octave higher, G to D), and 18 to 27 (D to A), also in the relationship of 2 to 3, together giving you the notes C, D, G, A: the open strings of the viola – or the cello, which has the same proportions but one octave lower. The violin open strings would have to add an E here which will triple all of the numbers in relation to the cello so 24:36:54:81.

Now I'd like to take one step back to involve all the consonances, and by consonance I just mean a harmonicity you can hear when tones are played together, consonance could also be any sounds from the piece you just heard, a ratio like 11/4 or 13/4. They're like dissonant consonances or structures of notes that have a periodic character, so they sound in tune, in a funny way, because they're also quite harsh. And that's part of the material that I find really interesting in just intonation: the *tuned dissonances*. To be able to do that, you have to depend on the relationships you can hear, and the framework that builds up from 2 and 3. So, what does that mean if you're thinking in the tempered system and you say, "OK, here's the tuning A, and that's 442hz," or whatever it is, and you set the tuning meter, and now it is reading 0 cents, then all of the notes that are in a perfect fifth relationship to each other will be deviated from 0 and from each other, exactly because *they are not tempered*. They won't sit on equal temperament because an equal tempered fifth is not producing 3/2. It's an interval that is too small. You can think of equal temperament as a sort of detuning of the Pythagorean frame, rather than the other way around, and this is exactly why it doesn't make any sense at all to use equal temperament as a basis for a tone system, like Schönberg wanted us to do. It's sloppy thinking and it kills harmony.

Q: What's the difference between 9:6 and 3:2?

A: As an interval it's exactly the same because the numbers reduce. The relationship is the same. 9:6 is 3 times 3:2, so in terms of absolute pitch it is one octave and a perfect fifth higher.

So that means that your perception of fundamental can change between G and C as well.

Well, here's the thing: your ear looks for the simplest interpretation of an interval, brings it (mathematically speaking) to lowest terms, which is why even if you play random combinations of chords out of a harmonic series, if it happens somewhere in the course of the piece that I just play G4 and G3 together, a 2/1, even if I am theoretically "in C", then G becomes the fundamental. It's not going to be heard as 12 and 6. The only way it's going to be 12 is if something is sounding or resounding the low C. To get a fundamental C, you need to hear a partial where the C is then implied so if you have 8:9, or 6:8:9:12, for example, of course you'll get C, because the fundamental is present, also as a combination tone, just above 16 Hz. If you combine harmonics 16 and 18 the fundamental would still be C, but in this case it would be an octave higher, about 33hz, since 16:18 reduces to 8:9. The fundamental will be like the low string of the five-string contrabass.

So, how does this information help us in terms of the problem we set out to investigate, the comma difference between the two tones called E? If we're going from C, we've got this E as 5 and its upper octaves: 10, 20, 40, 80 and so on; here on the viola open strings we have a C series that goes up in perfect fifths to its 27th partial, which is the tuning of the A string. What about the E natural, which tunes as a perfect fifth above A, this E natural here: on what harmonic number (thinking from C as fundamental) does *it* come about? Well, imagine every note here [points to open strings written up] has its harmonic series above it. So, the E natural in the D series is 8:9, it's the ninth harmonic of D. And D is itself the ninth of C. So what's the ninth of the ninth? 81. Or even simpler, from A it's the third harmonic. What's the third harmonic of 27? 81. Now if we take this E natural from the C harmonic series, it's number 5. Let's get it up in the same register, to compare the microtone, which is four octaves higher. So how does that work? Transposing up by one octave is multiplying by 2. So, 4 octaves is 16 times and harmonic 5 times 16 gives us 80. This is the syntonic comma, 81/80, the first microtone that comes from differences of differences. Now we're not talking exactly about differences of consonances, although it might be possibly heard that way.[10] It's a difference that is already slightly removed. So, you see, I play it by estimation. That's sort of like muscle and aural memory, to be made precise it requires some kind of intermediate structure, it requires the A and the G to play. Theoretically if you have A and G together, maybe in a wide voicing to soften the dissonances, you could maybe move the E accurately. But there's a reason why some of the smaller microtonal steps are less accessible, particularly the comma, because it's very difficult to find a way to move them in a melodic way, even though you could move between a consonant and a dissonant sound, it just doesn't have the same self-evident perceptual clarity as some other steps.

Now, this idea of moving through different consonances can give you all different kinds of sounds. There's the sounds like the ones here that I've shown, from the first five partials, but as you know from the harmonic series there's much more than that. The relationship from the seventh harmonic partial gives you a big whole tone, 7:8, and allows us to grasp what is actually meant by the term tritone, which means three wholetones right: 7:8:9:10? So for 7th harmonic intervals, I use the Tartini sign, proposed in the 1750's.[11] Tartini, a composer and violinist, discussed the natural seventh, which was already a topic throughout the 18th century, at length and proposed it could be successfully integrated in General-bass thinking.

We began discussing the comma to explain the principle of meantone tuning, remember the idea: simply that C and E stay where they are, and D goes down by half a comma, so it makes only one whole tone, not major (8 to 9), not minor (9 to 10) but a middle sized tone. That mean-sized whole tone is very close to the ratio 25:28. I can show you in just intonation how that comes about, it's interesting because we're getting towards a JI whole tone scale here.[12]

So, 7:8, 8:9, 9:10. That makes three whole tones of three different sizes. An *extension* of just intonation thinking to involve harmonics beyond five. Originally, theorists considered the only consonances to be Pythagorean, involving only the numbers 1, 2, 3, 4. *Senario* was Zarlino's term for the expansion of harmonic theory he proposed, to include consonant thirds and sixths, using all of the first *six* numbers 1, 2, 3, 4, 5, 6. Essentially this came to include all of the different multiples and composites made of the numbers 2, 3 and 5. So, mathematically any number that is a product of only 2, 3 and 5 would fit together according to perfect fifths and/or major thirds or sixths, or composites of the intervals in this logic. This ancient system, used in ragas of classical Indian music, which Harry Partch called *5-limit just*

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10 In my trio *Jean-Philippe Rameau* (2013), the syntonic comma interval is harmonized in three part counterpoint.

11 In Giuseppe Tartini's *Trattato Di Musica Secondo La Vera Scienza Dell' Armonia* (1754), the composer and violinist discusses difference tones and also invents a notation for the 7th harmonic, which he demonstrates in harmonized figured bass progressions . He comments that, unlike the dissonant seventh, which is generally required to descend in convential counterpoint, the consonant natural minor seventh can most beautifully be treated as a consonance, rising chromatically to move through the major third and resolve.

12 This explanantion was not completed during the lecture. Briefly, if the the pattern of intervals 8:9:10 is repeated twice among the notes C:D:E:F#:G#, so that D:E and F#:G# are minor wholetones, then the interval G#:Bb (with Bb tuned as 7°/C) will produce the ratio 25:28, and the resulting JI wholetone scale consists of the intervals 8:9, 9:10, 8:9, 9:10, 25:28, 7:8.

intonation, is what gets *extended* as higher primes, creating new *microtonally* varied intervals, are drawn into the mix.

If you involve 7, then you add a new dimension to the harmonic space. And as you extend into 11 and 13 you get into a realm whose harmonic sonorities have been less explored, except in some Partch music, or Ben Johnston music. And also, of course, in the music being written by Wolfgang von Schweinitz and in my own music, for example. But it's very well-known melodically in the tradition that grew out of the old Greek music, some of the different modal traditions, the Byzantine, Arabic, Persian traditions, and so on. There you have a progressive extension of the Pythagorean series to approximate ratios from higher primes, for example on fretted instruments tuned in fourths, and then theoretical writings that reinterpreted those finger positions as actual ratios of eleven. So, you have in the repertoire that we are gradually getting to know from around the world, and including what our small contribution in the West is to all of that, a lot of music that is already beginning to explore this realm of microtonally extended JI. But what is rather less common is the idea of integrating these tonal relationships in combinations which are more contrapuntal or polyphonic, bringing melody and harmony and rhythm based on these proportions together. That's an area that I've been looking at.

So how did I get into this? When I was in my late teens, starting my university studies, I wanted to do everything: compose, play the violin, study mathematics, and then I realized I couldn't do all of that at once, because I wasn't doing very well. So I ended up doing these things one after the other. I studied violin in Toronto and New York, and then afterwards I was playing in a string quartet and I realized I didn't want to keep doing that. I was getting more and more into composing, I was inspired by improvising, and first of all I worked with Malcolm Goldstein, and I got to know James Tenney. But the thing that was really driving me was my experience of these intervals, sort of these fine-tuning things I showed you with the two E naturals. I was very aware of these kind of paradoxes or differences, maybe part of it was because once when I played in a violin competition, the judge said "oh, you play so out of tune in all the double stops." I knew that I was playing a piece inspired by Persian music, calls to prayer, I was trying to get the intervals right, so I thought that she just didn't know anything, too narrow in her idea about intonation. And then I thought, why is it that the notes aren't *written* right – I was thinking about these kinds of things.

And I was living in St. John's, Newfoundland, kind of in the middle of the Atlantic Ocean. Looking for inspiration, I went to the library – no internet those days – and went to the section where there were music books, books by composers, browsing around. I was really into writing by composers, had been reading some John Cage texts and looking around for new stuff, and then this book popped out at me, Harry Partch's "Genesis of a Music." I just thought the title was kind of weird, a little bit religious, outrageous. And so I took it off the shelf and I looked inside – and in the beginning he starts in with this stuff about the mathematics of harmony, the ratios and intervals, and I thought, "wow, this is fantastic!" and got hooked. I sort of had an inkling of it by looking at how modes work, but the basic music training, the way we learn harmony in school, is completely backwards because we're learning it retrospectively, we understand it through the distorting lens of equal temperament and "chord progressions". The way that harmony actually evolved, the way that real texts about harmony were written in their time, historical texts by composers, not academics, describe *ratios*, they talk about the basic construction of frequencies, of tones, of relationships of sound, and then they say, "OK, on our tempered instruments we use this compromise, etc. so we can make counterpoint, instruments tune perfect intervals above the tempered bass line, etc." They may have various justifications to explain the system of their time, but at least it's acknowledged as such. I think it is a problem when a compromise gets justified and celebrated as a solution. Isn't it better to embrace the complexity, the paradoxes of sound?

So anyway, I found the Partch book and I started to try the stuff out myself. And the way that Partch approaches tone systems leads to a different way of thinking about them. I mean the tone systems he used for his instruments, he chose one so he could build instruments, and he always said, "this is one of many." Sometimes he had 43 tones, sometimes he had 39 tones, sometimes he had 57, he used however many notes he needed, because in the end, the tones are infinitely progressing in all directions. The principle that he discovered was that different categories of sounds are closely based on which prime harmonics combine into the harmony. So, if the backbone has the Pythagorean relationship of 3s, and then you explore the relationship of 5s, which have major thirds and can flow quite far, taken together with the 3s, or if you bring in the 7s or the 11s or the 13s, every new prime, which is therefore not built up from other pitches, has its own sounding potential, its own emotional and acoustical color. Even some of the prime numbers, which might normally only occur as summation tones in a large complex, these tones have a quality of their own, you can observe this in La Monte Young's music, for example.

So, my first approach was to determine, for myself: which of these things can I hear and which of these things can I not hear? How much of this theoretical information is actually possible to realize on an instrument without fixed pitches? I began exploring with the major thirds, with the sevenths, with the elevenths, thirteenths and so on, to try to distinguish sonorities, which I felt could be tuned within each category, which could be played, because I felt like if I was going to start writing for myself and for others to play on acoustic instruments, first of all it better be possible to hear it. Otherwise, I mean there's lots of microtonal music that you can try to play and it just sounds out of tune. And it's because it's not conceived on a sound basis, it's simply ideological. I like ratios, but I mean that stuff is essentially meaningless if it's not able to be transitioned into a practice.

Using electronic sounds is also a practice of course, there you can use any ratio you want, but then the question is: how do you hear it? I've heard exact electronic renditions of microtonal pieces that also sound "out of tune", and I am pretty sure it's not because I'm closed-minded about sounds. And that's in a way fundamental to the thing – can you can hear it for what it is, how much does it activate the underlying perceptual process that makes it harmonic?

[turns on projection from laptop with a list of tuneable intervals]

So, here you see some of these tuneable harmonic intervals, what I've done is written them up in relation to a fixed note. Now in principle, that fixed note could be below, or it could be above. An interval has two tones, and that interval could be placed anywhere in register. The first note is an A, that's your 1/1, in this particular diagram. The next one is a relationship of 7 to 8, so what it means is that I've taken this whole tone interval, 8/7, and activated it above the A, so that B natural is raised accordingly, the B natural is the fundamental and the A is its seventh. The 8:7 is the narrowest interval that one can still get a certain sense of, directly, without running into the critical band. I think it's perhaps possible, with training, to tune smaller intervals, but I'm not certain that we do it entirely in the same way, in terms of the sound.

It's quite beautiful, I think, that the next tuneable step above 8/7 is a microtonal melody that actually sounds like a harmony change, and it's a difference between two strong consonances. It's the characteristic La Monte Young interval, if any of you are fans of his music you'll recognize it from "The Well Tuned Piano." It's the difference between 8/7 and 7/6. The proportion of that melodic interval is 48:49, and using the measurement of cents it has a distance of 36 cents. What would that be as a fraction of a tone? A sixth tone.

The tunable intervals make relationships that form a network, these ratios are a starting point for producing sounds in JI and for moving from sound to sound. The piece which you heard at the beginning of my talk is built on almost all of these intervals. Basically, over the course of the piece, most the musicians are given the possibility of playing some of them. Either as a double stop on their own instrument or in relation to each other, they manage to seek out the sounds. And even if they don't manage to be perfectly in tune, I think they are able to suggest them, to evoke the fundamental relationships they imply. Remember, equal temperament is so completely out of tune, and yet in the historical music it was able to suggest to us harmonic worlds. I mean look at the incredible Romantic composers who went quite far writing crazy chords and modulations even inside the frame, or cage, of equal temperament.

So, when you approach the possibilities opened by this kind of precision of sound, I hope you can see you don't necessarily get a reactionary, neo-Romantic world, nor the opposite, a wilful Modernism: neither are necessary, nor excluded either, there's no reason for the material to determine an aesthetic or style, I believe it is transpersonal and quite limitless in the sense that it attempts to treat harmony *as it actually is perceived*. The point about microtonally extended just intonation, in my experience, is it's something which allows the sounds to flourish, and to be heard.

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