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Well temperament revisited:
two tunings for two keyboards
a quartertone apart
in extended JI

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for Johann Sebastian Bach

Well temperament revisited:
two tunings for two keyboards a quartertone apart in extended JI
by Marc Sabat

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Temperaments are sets of pitches in which some or all of the intervals are being *reinterpreted* as approximations of various just intervals.

In equal division temperaments, some interval, usually an octave, is divided into a number of equal parts, each with the same frequency ratio, usually irrational. In such systems each interval consists of *n* parts and is tuned the same in all transpositions. Intervals are standardised rather than varied, giving the impression that their irrational proportions are in some sense "correct". For intervals close to ratios, like the fifth in 12ED2, this property seems beneficial. It allows that particular tuning system to convincingly simulate the purity of melodic Pythagorean tuning while eliminating the problems of the Pythagorean comma. It does so, however, at the expense of the fundamental *sound* of beatless Pythagorean chords like 2:9:16, 6:8:9, 8:9:12, etc.

For other, less well represented ratios, 12ED2 simply replicates the same poorly tuned simulation in 12 transpositions. In this case, offering a *variety* of different approximations would offer a clear advantage. By allowing our ears to actually *hear* various shadings of intonation, their capacity to *perceive* contextually implied just intervals is stimulated. This, in turn, allows the available intervals to be more readily composed *harmonically* rather than *atonally*. Such an approach is common when writing for symphony orchestra, which combines many instruments of flexible tuning. In this context every interval is varied on a sliding scale navigated by chance and skill, determining the refinement and particular sonority of the ensemble. Only with the advent of MIDI guide tracks in the 1980s were orchestras forced to consistently reproduce 12ED2, blurring the resultant inharmonicity with continuous vibrato.

In well tempered tuning systems, transpositions of a given interval, though often *differing* from each other in terms of tuning, may still be perceived by the ear as the intended sounds, based on context. The historical motivation for this family of tunings with unequal steps was a desire to maintain the benefits of meantone temperament (pure thirds) in some keys while eliminating the very dissonant wolf fifth between G# and Eb. This was made possible by observing the similarity in size between the *Pythagorean comma* (531441/524288 or ca. 23.5 cents) – produced by traversing a series of 12 perfect fifths transposed within an octave – and the *Syntonic comma* (81/80 or ca. 21.5 cents) – the difference between four perfect fifths and a major third tuned as 5/4.

In the 17th and 18th centuries, various well temperaments were proposed (by Kirnberger, Werkmeister, Vallotti, Rameau, et al.) and to the present day there are advocates of specific solutions for particular repertoire (Lindley, Lehman, et al.). Revealing, however, is Johann Sebastian Bach's decision to not specify *any particular* well temperament for his 48 preludes and fugues. This is perhaps the case because of Bach's own experience, playing instruments in numerous coexisting tuning systems at various absolute pitch-heights. Whatever preferences he may have had when tuning his own harpsichord, Bach's

music demonstrates that *context* may convince the ear of harmonic relationships in spite of the specific choices of temperament. This point of view eventually led to the adoption and standardisation (in the 19th and 20th centuries) of 12ED2 at a standard Kammerton of 440 Hz. Ironically, the reduction of interval variety by a factor of 12 was soon followed by a move to new tone systems and atonality in new music. Over the course of the 20th century jazz harmony and the work of Scriabin and Messiaen, among others, sketched out the remaining unexplored *harmonic* possibilities of the 12-tone set.

As we move further into the 21st century, tonal applications of 12ED2 in contemporary classical, jazz, and popular music styles, though ubiquitous, seem more and more like tired clichés. Some composers have turned away from pitch to noise, conceptualism, or theatre, while others are pursuing microtonality and extended just intonation as a way to explore new harmonic sounds. At first glance, it seems if a single standardised alternative were to take the place of 12ED2 it would need to be a finer resolution equal temperament, such as 31ED2, 41ED2, 53ED2, or 72ED2, to name four of the best candidates for harmonic sound-combinations. Since none of these offer a *complete* approximation of tuneable sounds, however, none can be considered a complete tone system. At the same time, increasing the number of microtonal divisions greatly increases the ergonomic challenges of building and playing fixed-pitch instruments. For these reasons – too few intervals, too many notes – general adoption of a new form of equal temperament is an unlikely development.

The only really convincing choice as a general tone system for new harmonic music, then, is extended just intonation based on tuneable intervals. These, for the most part, are intervals derived from the first 19 harmonics of the series, including their multiples and combinations, and occasionally extending to include higher primes. Such a basis combines material shared by tonal music practices, old and new, from around the world – Chinese, Indian, Persian, Arabic, Greek, African, European, American,... – and at the same time opens fresh, as yet unexplored, horizons of sound and composition. This is possibly the most promising option for developing a more "universal" theoretical and practical basis for music.

Musicians would need to be able to reproduce or approximate just intervals within an aurally acceptable tolerance range depending on musical context. Fortunately, many of these sounds are what singers and musicians already hear as being "good" or "expressive" intonation – it simply requires extension and refinement of existing tonal skills along with an openness for new sonorities. Instruments need to be flexible enough to play *some* of these pitches, *more* of them, to combine with each other based on a common logic of extended JI to produce well-tuned harmonic sounds. In many cases (strings, brass, voice) this is already possible. For woodwinds, fixed-pitch keyboards and fretted instruments, more flexibility and diversity, not reductive standardisations, are needed. Different scordature, different Kammertons from period and regional practices, different frettings and temperaments already offer many possibilities to be explored and superimposed.

For instruments with a limited number of fixed pitches, the question becomes: which JI pitches might it make sense to tune, especially if the instrument, like a piano, cannot be adjusted before each piece in a concert. This opens the possibility of revisiting the old idea of well temperaments, and to consider how they might be *made for the most part out of just pitches*. This would combine compatibility with new JI music and old 12-note music. Thinking about this question led me to formulate two well tempered systems for two keyboard instruments a quartertone apart, which may be realised on pianos or other suitable instruments.

Sabat I was conceived in 2015 as a kind of "inverse Vallotti" well temperament, with six perfect fifths, and six fifths narrowed by ca. 1/6 comma. The seven white notes are tuned in Pythagorean diatonic, as a chain of 3/2 perfect fifths. The remaining pitches are tuned according to the 19th and 17th harmonics, creating slightly narrowed "fifths". G-Bb and F#-A are tuned as 19/16. C-Db and D#-E are tuned as 17/16. The remaining pitch, G#/Ab, is tuned to make a compromise between 17/16 and 18/17.

$$8 : 9 \quad \text{can be divided thus:}$$
$$16 : 17 : 18 \;=\; 272 : 289 : 306 \quad (\text{G} : \text{Ab} : \text{A})$$
$$\text{while} \quad 17 : 18 \;=\; 272 : 288 \quad (\text{G} : \text{G\#})$$
$$\text{so the difference between} \quad 17 : 18 \quad \text{and} \quad 16 : 17 \quad \text{is} \quad 288 : 289 \quad (\text{G\#} : \text{Ab})$$

$$\text{to split this difference harmonically it is divided thus:}$$
$$576 : 577 : 578 \quad (\text{G\#} : \text{G\#/Ab} : \text{Ab})$$

$$\text{G\#/Ab is tuned as 577 (a prime) by dividing} \quad \text{G} : \text{A} \quad \text{as}$$
$$(32*17 =) \quad 544 : 577 : 612 \quad (= 36*17)$$

Sabat II was conceived after working for a while with the earlier tuning and wishing for a variation, one which took advantage of the schisma (the difference between eight 3/2 fifths and a 5/4 major third) to produce 5-limit major thirds, like the very earliest Pythagorean keyboard tunings, and which, consequently, was easier to tune by ear. This led to a more extreme well temperament with somewhat spicier narrow "fifths" (ca. 1/3 comma narrower than just). In spite of these radical "fifths", this tuning sounds surprisingly convincing in performances of classical and new music.[1] The series of perfect 3/2 fifths is extended to include F#, and 5/4 major thirds F#-A# and Db-F are tuned. G-Ab and D#-E form 17/16 ratios, dividing the wholetones G : A and D : E in almost equal steps, harmonically and subharmonically.

For each of these two well temperaments, a complementary quartertone tuning has been conceived for a second piano, by employing the narrow fifth created by two stacked 11/9 intervals to move between undertonal and overtonal 11th harmonic sounds relating to the Pythagorean notes C G D A E. The continuity of fifths is made possible by combining 3, 5, 7, 11 and 13. In Sabat II, the major thirds of the main keyboard are extended to include natural 13th harmonics above and below the Db and A# respectively.

It is hoped that these new well temperaments offer viable alternatives to equal temperament that are suitable for performances of the old repertoire and at the same time for the creation of new music composed in microtonally extended just intonation.

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1   The first public performance of Sabat II was presented in Berlin by Thomas Nicholson at KM28, on 15 February 2019, in a program of music by Sabat, Beethoven, Nicholson, and Feldman.

"Well-Tempered" Extended JI Quartertone Tuning for Keyboard Instruments (Sabat I)

based on a Harmonic Space subset defined by the prime partials 3, 5, 7, 11, 13, 17, 19, and 577

Marc Sabat

Key (tuning repeats in all octaves; either ignore inharmonicities or adjust slightly to reduce beats in the 2 : 3 ratios by tuning unisons between 2nd and 3rd partials)

Tuning (Partch frequency-ratio pitch notation is normalized from D, cents-deviations from equal temperament are based on the Kammerton A = 0c)

| 577/408 | 18/17 | 19/12 | 32/27 | 16/9 | 4/3 | 1/1 | 3/2 | 9/8 | 27/16 | 24/19 | 17/9 | 577/408 |

-2   -3   -6.4   -7.8   -5.9   -3.9   -2   0   +2   +3.9   +2.5   -0.9   -2

577 : 864 = 699c

216 : 323 = 696.6c

256 : 171 = 698.6c

= 2 : 3 *
(577 : 576)
ca. -1/8 SC

= 2 : 3 *
(324 : 323)
ca. -1/4 SC

= 2 : 3 *
(513 : 512)
ca. -1/6 SC

All intervals are tuned symmetrically around D, the central pitch of staff notation and traditional keyboard design. The six fifths between white keys are tuned Pythagorean in frequency proportion 2 : 3 = 702c. The six remaining fifths are each made slightly smaller, by approximately 1/6, 1/4 and 1/8 of a Syntonic Comma, producing a "circle" of fifths. The "chromatic" keys divide the Pythagorean wholetones into semitones 16 : 17 (C - Db and E - D#), 17 : 18 (Db - D and D# - D), 18 : 19 (A - Bb and G - F#) and 76 : 81 (Bb - B and F# - F).

256 : 171 = 698.6c

216 : 323 = 696.6c

1156 : 1731 = 699c

= 2 : 3 *
(513 : 512)
ca. -1/6 SC

= 2 : 3 *
(324 : 323)
ca. -1/4 SC

= 2 : 3 *
(578 : 577)
ca. -1/8 SC

The ratio for G#/Ab is determined by dividing the wholetone G : A arithmetically in the frequency proportion 16 : 17 : 18 (obtaining an Ab) and harmonically in the frequency proportion 272 (=17·16) : 288 (=18·16) : 306 (=18·17) (obtaining a G#). The ratio G# : Ab, 288 : 289 (=17·17) is divided in the proportion 576 : 577 : 578. The intermediate pitch 577 produces a diminished fifth D - Ab of 600.003c, and a tritone Ab - D of 599.997c.

A second keyboard tuned as below augments the well-tempered tuning with quartertones.

385 : 576 = 697.5c

117 : 175 = 697c

225 : 338 = 704.5c

= 2 : 3 *
(385 : 384)
ca. -1/5 SC

= 2 : 3 *
(351 : 350)
ca. -1/4 SC

= 2 : 3 *
(675 : 676)
ca. +1/8 SC

| 18/11 | 11/9 | 11/6 | 11/8 | 33/32 | 54/35 | 15/13 | 26/15 |

-49.4   +45.5   +47.4   +49.4   +51.3   +48.8   +45.8   -49.7

175 : 117 = 697c

576 : 385 = 697.5c

81 : 121 = 694.8c

= 2 : 3 *
(243 : 242)
ca. -1/3 SC

| 26/15 | 35/27 | 64/33 | 16/11 | 12/11 | 18/11 | 11/9 |

-49.7   -52.7   -55.2   -53.3   -51.3   -49.4   +45.5

Berlin, 7 December 2015 / 14 February 2016

"Well-Tempered" Extended JI Quartertone Tuning for Keyboard Instruments (Sabat II)

based on a Harmonic Space subset defined by the prime partials 3, 5, 7, 11, 13, and 17

Marc Sabat

Key (tuning repeats in all octaves; either ignore inharmonicities or adjust slightly to reduce beats in the 2 : 3 ratios by tuning unisons between 2nd and 3rd partials)

Tuning (Partch frequency-ratio pitch notation is normalized from A, cents-deviations from equal temperament are based on the Kammerton A = 0c)

| 17/9 | 24/17 | 135/128 | 128/81 | 32/27 | 16/9 | 4/3 | 1/1 | 3/2 | 9/8 | 27/16 | 512/405 | 17/9 |

+5.9 +1.0
+5.9
+3.9
+2
-7.8
0
-2
-3.9
-5.9
-7.8
-3.0
+1.0

289 : 432
= 696.0c
= 2 : 3 *
(289 : 288)
ca. -1/4 PC

512 : 765
= 695.2c
= 2 : 3 *
(256 : 255)
ca. -1/3 SC

10935 : 16384
= 700.0c
= 2 : 3 *
(32805 : 32768)
ca. -1/12 PC

All intervals are tuned symmetrically around the D-A axis. Seven fifths between white keys F-C-G-D-A-E-B-F♯ are tuned Pythagorean in frequency proportion 2 : 3 = 702c. Db-F and F♯-A♯ are tuned in the ratio 4 : 5, so that the diminished sixths A♯-F and F♯-Db are each narrower by one schisma. G-Ab and D♯-E are tuned in the ratio 16 : 17, so that the fifths D♯-A♯ and Db-Ab are both narrowed by a septendecimal schisma. The remaining doubly-augmented fourth Ab-D♯ is tuned 289 : 432.

10935 : 16384
= 700.0c
= 2 : 3 *
(32805 : 32768)
ca. -1/12 PC

512 : 765
= 695.2c
= 2 : 3 *
(256 : 255)
ca. -1/3 SC

A second keyboard tuned as below augments the well-tempered tuning with quartertones.

81 : 121
= 694.8c
= 2 : 3 *
(243 : 242)
ca. -1/3 SC

385 : 576
= 697.5c
= 2 : 3 *
(385 : 384)
ca. -1/5 PC

243 : 364
= 699.6c
= 2 : 3 *
(729 : 728)
ca. -1/10 PC

1352 : 2025
= 699.4c
= 2 : 3 *
(676 : 675)
ca. -1/9 PC

| 12/11 | 44/27 | 11/9 | 11/6 | 48/35 | 416/405 | 208/135 | 15/13 |

+45.5 +47.4 +49.4 +46.8 A+46.4 E+48.3 +47.7
-49.4

243 : 364
= 699.6c

385 : 576
= 697.5c

81 : 121
= 694.8c

| 15/13 | 45/26 | 135/104 | 35/18 | 16/11 | 12/11 | 44/27 |

C-52.3 G-50.3 D-48.3 -48.8 -51.3 -49.4 +45.5

Berlin, 19 February 2019