Three Crystal Growth Algorithms in 23-limit constrained Harmonic Space

by Marc Sabat

PLAINSOUND MUSIC EDITION

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in 23-limit constrained Harmonic Space

by Marc Sabat after JAMES TENNEY

for Jim with love – thank you

The following three tables of 190 pitches develop some ideas suggested in James Tenney's paper "On 'Crystal Growth' in Harmonic Space" (1993-98). To generate my crystals I began by looking at his computer programs, written in BASIC, and then coded an extension of his algorithm in C++. The starting premises remained the same – to be chosen each new point must

(1.) be directly connected to a point of the crystal and

(2.) produce the least possible HD-sum (sum of harmonic distances to all other points of the crystal).

In harmonic space (HS), each pitch is represented by co-ordinates, which are exponents of the prime factors of its frequency ratio. As the number of primes is infinite, HS theoretically has infinite dimension; for musical applications, it is constrained by specifying a finite number of prime axes. For example, the classical Riemann tone-lattice is a subset of (2,3,5)-HS, in which the frequency ratio 3/2 ($= 2^{-1} \cdot 3^{1} \cdot 5^{0}$) is represented by the pitch-point (-1,1,0). As Tenney's crystals show (see Figure 10a/b), any HS includes points outside the range of musical perception. This suggests the possibility to add an additional constraint – *pitch-height range*.

To define connection in a pitch-height constrained HS, it is useful to borrow an idea from the construction of *pitch-class projection space* (PS). In music we often think of "octave-equivalent pitch-classes"; similarly in projection spaces *the 2-dimension is ignored*. Thus, if A is taken as the pitch-point 1/1, it is sufficient to write up *any* E as 3/2 – the ratio then refers equally well to HS points 3/1, 3/4, 3/8 etc., sharing the same pitch-class but in different octaves.

In PS a connection is said to exist between two pitch-classes *if a connection exists at some octave transposition* of their pitch-points in HS. For example, 21/16 is connected to 3/2 in PS, because in HS 21/16 is an octave transposition of the pitch-point 21/2, which is connected to the HS point 3/2 in HS by taking one step in the 7-dimension ($3 \cdot 7 = 21$). From this idea we can generalize – *two points in pitch-height constrained HS are connected if and only if they are either connected or identical at some octave-transposition.* Under this definition, during the process of crystal-growth new *candidates* for points are selected by

(1.) finding a connected pitch-point in HS, then

(2.) considering all octave-transpositions of that pitch which fall within the available range (whilst considering the HD value of each octave transposition taken).

The algorithm chooses a candidate with least HD sum.

In the first paragraph of his article Tenney writes: "There will be frequent 'branch-points', where two or more ratios have equally minimal HD sums, and here the choice might be random." When a random choice is implemented, the crystals tend to wander or drift in harmonic space, losing their orientation around the origin (1/1). Wolfgang von Schweinitz suggested a refinement –

(1.) if two points have the same least HD sum, choose the one with a simpler ratio (smaller HD value);

(2.) if both points have the same HD make a random choice.

Without this test, emergent crystals may have a *geometrically* (and thus *musically)* similar form, but their constituent ratios will be more complex. Namely, the modified algorithm makes sure that crystal growth remains *centered* around 1/1.

In the three tables that follow, I have generated pitches in 23-limit Harmonic Space, because for each of the first nine prime numbers:

(2, 3, 5, 7, 11, 13, 17, 19, 23)

a *tuneable interval* exists:

(2/1, 3/2, 5/4, 7/4, 11/4, 13/4, 17/4, 19/4, 23/8).

This means that the resulting notes may be realized by ear, if musical *tuneable paths* between them are appropriately composed. The first table is in projection space, the second and third are in harmonic space constrained to the octave between 1/1 and 2/1. The second table does not include the pitch 2/1, in constrast to the third, which does.

I find it a fascinating musical meditation to contemplate the differences in the results of these three processes – PS favors the emergence of melodic *modes* and *scales*, whilst the pitch-height constrained HS favors *chord* formation.

The list of pitches ends more or less at the point where I felt I could no longer find their names!

Valencia, California 23.01.2007

Pitch-class Projection Space

Harmonic Space Co-ordinates | Prime Limit | Ratio | Interval
dimensions: 3,5,7,11,13,17,19,23

1: (0,0,0,0,0,0,0,0) | 2- | 1/1 | *unison*

2: (1,0,0,0,0,0,0,0) | 3- | 3/2 | *fifth*
3: (-1,0,0,0,0,0,0,0) | 3- | 4/3 | *fourth*
4: (2,0,0,0,0,0,0,0) | 3- | 9/8 | *major wholetone (dominant wholetone)*

5: (0,1,0,0,0,0,0,0) | 5- | 5/4 | *ptolemaic major third*
6: (1,1,0,0,0,0,0,0) | 5- | 15/8 | *ptolemaic major seventh*
7: (-1,1,0,0,0,0,0,0) | 5- | 5/3 | *ptolemaic major sixth*
8: (2,1,0,0,0,0,0,0) | 5- | 45/32 | *ptolemaic tritone*
9: (1,-1,0,0,0,0,0,0) | 5- | 6/5 | *ptolemaic minor third*
10: (0,-1,0,0,0,0,0,0) | 5- | 8/5 | *ptolemaic minor sixth*
11: (2,-1,0,0,0,0,0,0) | 5- | 9/5 | *ptolemaic minor seventh*
12: (-1,-1,0,0,0,0,0,0) | 5- | 16/15 | *major diatonic semitone*
13: (-2,0,0,0,0,0,0,0) | 3- | 16/9 | *pythagorean minor seventh*
14: (3,0,0,0,0,0,0,0) | 3- | 27/16 | *pythagorean major sixth*

15: (0,0,1,0,0,0,0,0) | 7- | 7/4 | *natural seventh*
16: (1,0,1,0,0,0,0,0) | 7- | 21/16 | *septimal narrow fourth*
17: (0,0,-1,0,0,0,0,0) | 7- | 8/7 | *septimal wholetone*
18: (1,0,-1,0,0,0,0,0) | 7- | 12/7 | *septimal major sixth*
19: (-1,0,1,0,0,0,0,0) | 7- | 7/6 | *septimal minor third*
20: (2,0,1,0,0,0,0,0) | 7- | 63/32 | *septimal narrow octave*
21: (2,0,-1,0,0,0,0,0) | 7- | 9/7 | *septimal major third*
22: (-1,0,-1,0,0,0,0,0) | 7- | 32/21 | *septimal wide fifth*
23: (0,-1,1,0,0,0,0,0) | 7- | 7/5 | *septimal diminished fifth*
24: (1,-1,1,0,0,0,0,0) | 7- | 21/20 | *minor diatonic semitone*
25: (0,1,1,0,0,0,0,0) | 7- | 35/32 | *septimal neutral second*
26: (1,1,1,0,0,0,0,0) | 7- | 105/64 | *septimal neutral sixth*
27: (-1,-1,1,0,0,0,0,0) | 7- | 28/15 | *septimal diminished octave*
28: (-1,1,1,0,0,0,0,0) | 7- | 35/24 | *septimal half-diminished fifth*
29: (-2,0,1,0,0,0,0,0) | 7- | 14/9 | *septimal minor sixth*
30: (2,-1,1,0,0,0,0,0) | 7- | 63/40 | *narrow septimal minor sixth*
31: (2,1,1,0,0,0,0,0) | 7- | 315/256 |
32: (3,0,1,0,0,0,0,0) | 7- | 189/128 |
33: (0,0,2,0,0,0,0,0) | 7- | 49/32 |
34: (1,0,2,0,0,0,0,0) | 7- | 147/128 |
35: (-2,1,0,0,0,0,0,0) | 5- | 10/9 | *minor wholetone (subdominant wholetone)*

36: (-2,-1,0,0,0,0,0,0) | 5- | 64/45 | *ptolemaic diminished fifth*
37: (-2,1,1,0,0,0,0,0) | 7- | 35/18 | *septimal half-diminished octave*
38: (-2,-1,1,0,0,0,0,0) | 7- | 56/45 |
39: (-1,0,2,0,0,0,0,0) | 7- | 49/48 | *major septimal sixthtone*
40: (-3,0,0,0,0,0,0,0) | 3- | 32/27 | *pythagorean minor third*
41: (-3,0,1,0,0,0,0,0) | 7- | 28/27 | *septimal thirdtone*
42: (0,1,-1,0,0,0,0,0) | 7- | 10/7 | *septimal tritone*
43: (0,-1,-1,0,0,0,0,0) | 7- | 64/35 | *septimal neutral seventh*
44: (-2,0,-1,0,0,0,0,0) | 7- | 64/63 | *septimal comma*
45: (1,1,-1,0,0,0,0,0) | 7- | 15/14 | *major chromatic semitone*
46: (-1,1,-1,0,0,0,0,0) | 7- | 40/21 | *large septimal major seventh*
47: (1,-1,-1,0,0,0,0,0) | 7- | 48/35 | *septimal half-augmented fourth*
48: (-1,-1,-1,0,0,0,0,0) | 7- | 128/105 | *septimal neutral third*
49: (0,2,0,0,0,0,0,0) | 5- | 25/16 | *ptolemaic augmented fifth*
50: (0,-2,0,0,0,0,0,0) | 5- | 32/25 | *ptolemaic diminished fourth*

51: (0,0,0,1,0,0,0,0) | 11- | 11/8 | *undecimal half-augmented fourth*
52: (0,0,0,-1,0,0,0,0) | 11- | 16/11 | *undecimal half-diminished fifth*

53: (0,0,0,0,1,0,0,0) | 13- | 13/8 | *tridecimal neutral sixth*
54: (0,0,0,0,-1,0,0,0) | 13- | 16/13 | *major tridecimal neutral third*
55: (-1,0,0,1,0,0,0,0) | 11- | 11/6 | *large undecimal neutral seventh*
56: (1,0,0,1,0,0,0,0) | 11- | 33/32 | *undecimal quartertone*
57: (1,0,0,-1,0,0,0,0) | 11- | 12/11 | *small undecimal 3/4-tone*
58: (-1,0,0,-1,0,0,0,0) | 11- | 64/33 | *undecimal narrow octave*
59: (-1,2,0,0,0,0,0,0) | 5- | 25/24 | *minor chromatic semitone*
60: (1,2,0,0,0,0,0,0) | 5- | 75/64 | *ptolemaic augmented second*
61: (1,-2,0,0,0,0,0,0) | 5- | 48/25 | *large diminished octave*
62: (-1,-2,0,0,0,0,0,0) | 5- | 128/75 | *ptolemaic diminished seventh*
63: (-1,0,0,0,1,0,0,0) | 13- | 13/12 | *large tridecimal neutral second*
64: (1,0,0,0,1,0,0,0) | 13- | 39/32 | *minor tridecimal neutral third*
65: (1,0,0,0,-1,0,0,0) | 13- | 24/13 | *tridecimal neutral seventh*
66: (-1,0,0,0,-1,0,0,0) | 13- | 64/39 |

67: (0,0,0,0,0,1,0,0) | 17- | 17/16 | *large 17-limit semitone*
68: (0,0,0,0,0,-1,0,0) | 17- | 32/17 |

69: (0,0,0,0,0,0,1,0) | 19- | 19/16 | *19-limit minor third*
70: (0,0,0,0,0,0,-1,0) | 19- | 32/19 |
71: (0,0,1,-1,0,0,0,0) | 11- | 14/11 | *undecimal diminished fourth*
72: (0,0,1,1,0,0,0,0) | 11- | 77/64 |
73: (0,-1,0,1,0,0,0,0) | 11- | 11/10 | *large undecimal 3/4-tone*
74: (0,1,0,1,0,0,0,0) | 11- | 55/32 | *undecimal large major sixth*

75: (0,1,0,-1,0,0,0,0) | 11- | 20/11 | *small undecimal neutral seventh*
76: (0,-1,0,-1,0,0,0,0) | 11- | 64/55 |
77: (3,-1,0,0,0,0,0,0) | 5- | 27/20 | *ptolemaic wide fourth*
78: (3,1,0,0,0,0,0,0) | 5- | 135/128 | *major limma*
79: (1,0,0,0,0,-1,0,0) | 17- | 24/17 | *17-limit tritone*
80: (1,0,0,0,0,1,0,0) | 17- | 51/32 |
81: (-1,0,0,0,0,1,0,0) | 17- | 17/12 | *17-limit diminished fifth*
82: (-1,0,0,0,0,-1,0,0) | 17- | 64/51 |

83: (0,0,0,0,0,0,0,1) | 23- | 23/16 | *23-limit tritone*
84: (0,0,0,0,0,0,0,-1) | 23- | 32/23 |
85: (-3,1,0,0,0,0,0,0) | 5- | 40/27 | *ptolemaic narrow fifth*
86: (-3,-1,0,0,0,0,0,0) | 5- | 256/135 |
87: (-1,0,0,0,0,0,1,0) | 19- | 19/12 | *19-limit minor sixth*
88: (1,0,0,0,0,0,1,0) | 19- | 57/32 |
89: (1,0,0,0,0,0,-1,0) | 19- | 24/19 |
90: (-1,0,0,0,0,0,-1,0) | 19- | 64/57 |
91: (-2,0,0,1,0,0,0,0) | 11- | 11/9 | *undecimal neutral third*
92: (2,0,0,1,0,0,0,0) | 11- | 99/64 |
93: (-1,-1,0,1,0,0,0,0) | 11- | 22/15 | *undecimal diminished fifth*
94: (1,-1,0,1,0,0,0,0) | 11- | 33/20 |
95: (-1,1,0,1,0,0,0,0) | 11- | 55/48 |
96: (1,1,0,1,0,0,0,0) | 11- | 165/128 |
97: (-1,0,1,1,0,0,0,0) | 11- | 77/48 |
98: (1,0,1,1,0,0,0,0) | 11- | 231/128 |
99: (0,0,-1,1,0,0,0,0) | 11- | 11/7 | *undecimal augmented fifth*
100: (2,0,0,-1,0,0,0,0) | 11- | 18/11 | *undecimal neutral sixth*
101: (-2,0,0,-1,0,0,0,0) | 11- | 128/99 |
102: (4,0,0,0,0,0,0,0) | 3- | 81/64 | *pythagorean major third*
103: (1,1,0,-1,0,0,0,0) | 11- | 15/11 | *undecimal augmented fourth*
104: (1,-1,0,-1,0,0,0,0) | 11- | 96/55 |
105: (-1,1,0,-1,0,0,0,0) | 11- | 40/33 |
106: (-1,-1,0,-1,0,0,0,0) | 11- | 256/165 |
107: (1,0,1,-1,0,0,0,0) | 11- | 21/11 | *undecimal diminished octave*
108: (-1,0,1,-1,0,0,0,0) | 11- | 56/33 |
109: (-4,0,0,0,0,0,0,0) | 3- | 128/81 | *comma-diminished octave*
110: (0,0,-1,-1,0,0,0,0) | 11- | 128/77 |
111: (0,-1,0,0,1,0,0,0) | 13- | 13/10 | *tridecimal half-diminished fourth*
112: (0,1,0,0,1,0,0,0) | 13- | 65/64 | *large tridecimal eighthtone*
113: (0,1,0,0,-1,0,0,0) | 13- | 20/13 | *tridecimal half-augmented fifth*
114: (0,-1,0,0,-1,0,0,0) | 13- | 128/65 | *tridecimal diminished octave*
115: (0,0,1,0,-1,0,0,0) | 13- | 14/13 | *small tridecimal neutral second*
116: (0,0,1,0,1,0,0,0) | 13- | 91/64 |

117: (0,-2,1,0,0,0,0,0) | 7- | 28/25 | *septimal diminished third*
118: (0,2,1,0,0,0,0,0) | 7- | 175/128 |
119: (-1,0,0,0,0,0,0,1) | 23- | 23/12 | *23-limit major seventh*
120: (1,0,0,0,0,0,0,1) | 23- | 69/64 |
121: (1,0,0,0,0,0,0,-1) | 23- | 24/23 | *23-limit semitone*
122: (-1,0,0,0,0,0,0,-1) | 23- | 128/69 |
123: (0,0,-2,0,0,0,0,0) | 7- | 64/49 |
124: (-2,0,0,0,1,0,0,0) | 13- | 13/9 | *tridecimal diminished fifth*
125: (2,0,0,0,1,0,0,0) | 13- | 117/64 |
126: (-1,-1,0,0,1,0,0,0) | 13- | 26/15 |
127: (1,-1,0,0,1,0,0,0) | 13- | 39/20 |
128: (-1,1,0,0,1,0,0,0) | 13- | 65/48 |
129: (1,1,0,0,1,0,0,0) | 13- | 195/128 |
130: (-1,0,1,0,1,0,0,0) | 13- | 91/48 |
131: (1,0,1,0,1,0,0,0) | 13- | 273/256 |
132: (2,0,0,0,-1,0,0,0) | 13- | 18/13 | *tridecimal augmented fourth*
133: (-2,0,0,0,-1,0,0,0) | 13- | 128/117 |
134: (1,1,0,0,-1,0,0,0) | 13- | 15/13 | *tridecimal augmented second*
135: (-1,1,0,0,-1,0,0,0) | 13- | 40/39 | *tridecimal quartertone*
136: (1,-1,0,0,-1,0,0,0) | 13- | 96/65 |
137: (-1,-1,0,0,-1,0,0,0) | 13- | 256/195 |
138: (1,0,1,0,-1,0,0,0) | 13- | 21/13 |
139: (-1,0,1,0,-1,0,0,0) | 13- | 56/39 |
140: (-2,2,0,0,0,0,0,0) | 5- | 25/18 | *Rameau's tritone*
141: (2,2,0,0,0,0,0,0) | 5- | 225/128 | *ptolemaic augmented sixth*
142: (2,1,-1,0,0,0,0,0) | 7- | 45/28 |
143: (-2,1,-1,0,0,0,0,0) | 7- | 80/63 |
144: (0,1,0,0,0,-1,0,0) | 17- | 20/17 | *17-limit augmented second*
145: (0,1,0,0,0,1,0,0) | 17- | 85/64 |
146: (2,-1,-1,0,0,0,0,0) | 7- | 36/35 | *septimal quartertone*
147: (2,-2,0,0,0,0,0,0) | 5- | 36/25 | *Rameau's false fifth*
148: (-2,-1,-1,0,0,0,0,0) | 7- | 512/315 |
149: (-2,-2,0,0,0,0,0,0) | 5- | 256/225 | *ptolemaic diminished third*
150: (0,-1,0,0,0,1,0,0) | 17- | 17/10 | *17-limit dimished seventh*
151: (0,-1,0,0,0,-1,0,0) | 17- | 128/85 |
152: (0,0,-1,0,1,0,0,0) | 13- | 13/7 | *tridecimal major seventh*
153: (0,0,-1,0,-1,0,0,0) | 13- | 128/91 |
154: (3,0,-1,0,0,0,0,0) | 7- | 27/14 | *small septimal major seventh*
155: (-3,0,-1,0,0,0,0,0) | 7- | 256/189 |
156: (1,0,-2,0,0,0,0,0) | 7- | 96/49 |
157: (-1,0,-2,0,0,0,0,0) | 7- | 256/147 |
158: (-1,0,-1,1,0,0,0,0) | 11- | 22/21 | *undecimal semitone*
159: (1,0,-1,1,0,0,0,0) | 11- | 33/28 |

160: (1,0,-1,-1,0,0,0,0) | 11- | 96/77 |
161: (-1,0,-1,-1,0,0,0,0) | 11- | 256/231 |
162: (0,2,-1,0,0,0,0,0) | 7- | 25/14 | *septimal augmented sixth*
163: (0,-2,-1,0,0,0,0,0) | 7- | 256/175 |
164: (-1,0,-1,0,1,0,0,0) | 13- | 26/21 |
165: (1,0,-1,0,1,0,0,0) | 13- | 39/28 |
166: (1,0,-1,0,-1,0,0,0) | 13- | 96/91 |
167: (-1,0,-1,0,-1,0,0,0) | 13- | 512/273 |
168: (0,-1,0,0,0,0,1,0) | 19- | 19/10 | *19-limit diminished octave*
169: (0,1,0,0,0,0,1,0) | 19- | 95/64 |
170: (0,1,0,0,0,0,-1,0) | 19- | 20/19 | *small 19-limit semitone*
171: (0,-1,0,0,0,0,-1,0) | 19- | 128/95 |
172: (-2,0,0,0,0,1,0,0) | 17- | 17/9 | *17-limit diminished octave*
173: (2,0,0,0,0,1,0,0) | 17- | 153/128 |
174: (0,0,-1,0,0,1,0,0) | 17- | 17/14 | *17-limit minor third*
175: (0,0,1,0,0,1,0,0) | 17- | 119/64 |
176: (-1,-1,0,0,0,1,0,0) | 17- | 17/15 | *17-limit diminished third*
177: (1,-1,0,0,0,1,0,0) | 17- | 51/40 |
178: (-1,1,0,0,0,1,0,0) | 17- | 85/48 |
179: (1,1,0,0,0,1,0,0) | 17- | 255/128 |
180: (2,0,0,0,0,-1,0,0) | 17- | 18/17 | *Galileo's 99¢ semitone*
181: (-2,0,0,0,0,-1,0,0) | 17- | 256/153 |
182: (1,1,0,0,0,-1,0,0) | 17- | 30/17 |
183: (-1,1,0,0,0,-1,0,0) | 17- | 80/51 |
184: (1,-1,0,0,0,-1,0,0) | 17- | 96/85 |
185: (-1,-1,0,0,0,-1,0,0) | 17- | 256/255 |
186: (0,0,1,0,0,-1,0,0) | 17- | 28/17 |
187: (0,0,-1,0,0,-1,0,0) | 17- | 128/119 |
188: (-1,2,-1,0,0,0,0,0) | 7- | 25/21 | *septimal augmented second*
189: (1,2,-1,0,0,0,0,0) | 7- | 75/56 |
190: (-1,2,1,0,0,0,0,0) | 7- | 175/96 |

Harmonic Space constrained by 1/1 ≤ P < 2/1

Harmonic Space Co-ordinates | Prime Limit | Ratio | Interval
dimensions: 2,3,5,7,11,13,17,19,23

1: (0,0,0,0,0,0,0,0,0) | 2- | 1/1 | *unison*

2: (-1,1,0,0,0,0,0,0,0) | 3- | 3/2 | *fifth*

3: (-2,0,1,0,0,0,0,0,0) | 5- | 5/4 | *ptolemaic major third*
4: (-3,1,1,0,0,0,0,0,0) | 5- | 15/8 | *ptolemaic major seventh*
5: (0,-1,1,0,0,0,0,0,0) | 5- | 5/3 | *ptolemaic major sixth*

6: (-1,1,1,-1,0,0,0,0,0) | 7- | 15/14 | *major chromatic semitone*
7: (1,0,1,-1,0,0,0,0,0) | 7- | 10/7 | *septimal tritone*
8: (-2,2,1,-1,0,0,0,0,0) | 7- | 45/28 |
9: (-1,0,2,-1,0,0,0,0,0) | 7- | 25/14 | *septimal augmented sixth*
10: (0,2,0,-1,0,0,0,0,0) | 7- | 9/7 | *septimal major third*
11: (2,1,0,-1,0,0,0,0,0) | 7- | 12/7 | *septimal major sixth*
12: (-3,2,0,0,0,0,0,0,0) | 3- | 9/8 | *major wholetone (dominant wholetone)*
13: (-1,3,0,-1,0,0,0,0,0) | 7- | 27/14 | *small septimal major seventh*
14: (-3,1,2,-1,0,0,0,0,0) | 7- | 75/56 |
15: (3,0,0,-1,0,0,0,0,0) | 7- | 8/7 | *septimal wholetone*
16: (0,-1,2,-1,0,0,0,0,0) | 7- | 25/21 | *septimal augmented second*
17: (3,-1,1,-1,0,0,0,0,0) | 7- | 40/21 | *large septimal major seventh*
18: (2,-1,0,0,0,0,0,0,0) | 3- | 4/3 | *fourth*
19: (1,-2,1,0,0,0,0,0,0) | 5- | 10/9 | *minor wholetone (subdominant wholetone)*
20: (1,1,-1,0,0,0,0,0,0) | 5- | 6/5 | *ptolemaic minor third*
21: (0,2,-1,0,0,0,0,0,0) | 5- | 9/5 | *ptolemaic minor seventh*

22: (0,1,1,0,-1,0,0,0,0) | 11- | 15/11 | *undecimal augmented fourth*

23: (0,1,1,0,0,-1,0,0,0) | 13- | 15/13 | *tridecimal augmented second*
24: (2,1,1,-2,0,0,0,0,0) | 7- | 60/49 |
25: (1,2,1,-2,0,0,0,0,0) | 7- | 90/49 |
26: (2,2,-1,-1,0,0,0,0,0) | 7- | 36/35 | *septimal quartertone*
27: (1,2,1,-1,-1,0,0,0,0) | 11- | 90/77 |
28: (0,1,2,-2,0,0,0,0,0) | 7- | 75/49 |
29: (0,0,0,-1,1,0,0,0,0) | 11- | 11/7 | *undecimal augmented fifth*
30: (0,0,0,-1,0,1,0,0,0) | 13- | 13/7 | *tridecimal major seventh*
31: (1,0,2,-2,0,0,0,0,0) | 7- | 50/49 | *minor septimal sixthtone*
32: (-2,0,1,-1,1,0,0,0,0) | 11- | 55/28 |
33: (-2,1,0,-1,1,0,0,0,0) | 11- | 33/28 |

34: (-1,-1,1,-1,1,0,0,0,0) | 11- | 55/42 |
35: (1,-1,0,-1,1,0,0,0,0) | 11- | 22/21 | *undecimal semitone*
36: (-2,1,0,-1,0,1,0,0,0) | 13- | 39/28 |
37: (-1,-1,1,-1,0,1,0,0,0) | 13- | 65/42 |
38: (1,-1,0,-1,0,1,0,0,0) | 13- | 26/21 |

39: (-1,0,0,-1,0,0,1,0,0) | 17- | 17/14 | *17-limit minor third*
40: (-3,0,1,-1,0,1,0,0,0) | 13- | 65/56 |

41: (-1,0,0,-1,0,0,0,1,0) | 19- | 19/14 | *19-limit wide fourth*
42: (-2,1,0,-1,0,0,1,0,0) | 17- | 51/28 |

43: (-1,0,0,-1,0,0,0,0,1) | 23- | 23/14 |
44: (-2,0,0,1,0,0,0,0,0) | 7- | 7/4 | *natural seventh*
45: (-1,-1,0,0,1,0,0,0,0) | 11- | 11/6 | *large undecimal neutral seventh*
46: (-1,-1,0,1,0,0,0,0,0) | 7- | 7/6 | *septimal minor third*
47: (5,-1,0,-1,0,0,0,0,0) | 7- | 32/21 | *septimal wide fifth*
48: (1,-1,0,-1,0,0,1,0,0) | 17- | 34/21 |
49: (3,0,-1,0,0,0,0,0,0) | 5- | 8/5 | *ptolemaic minor sixth*
50: (4,1,-1,-1,0,0,0,0,0) | 7- | 48/35 | *septimal half-augmented fourth*
51: (1,3,-1,-1,0,0,0,0,0) | 7- | 54/35 |
52: (1,1,-1,-1,1,0,0,0,0) | 11- | 66/35 |
53: (0,1,-1,-1,0,1,0,0,0) | 13- | 39/35 |
54: (2,0,-1,-1,1,0,0,0,0) | 11- | 44/35 |
55: (1,-1,0,-1,0,0,0,1,0) | 19- | 38/21 |
56: (0,-1,0,-1,0,0,0,0,1) | 23- | 23/21 |
57: (1,1,0,-2,1,0,0,0,0) | 11- | 66/49 |
58: (3,2,0,-2,0,0,0,0,0) | 7- | 72/49 |
59: (1,3,0,-2,0,0,0,0,0) | 7- | 54/49 |
60: (0,0,1,-2,1,0,0,0,0) | 11- | 55/49 |
61: (4,0,1,-2,0,0,0,0,0) | 7- | 80/49 |
62: (-1,1,1,-2,1,0,0,0,0) | 11- | 165/98 |
63: (-3,2,0,-1,1,0,0,0,0) | 11- | 99/56 |
64: (-1,2,-1,-1,1,0,0,0,0) | 11- | 99/70 |
65: (-1,3,1,-2,0,0,0,0,0) | 7- | 135/98 |
66: (-1,2,0,-2,1,0,0,0,0) | 11- | 99/98 |
67: (1,1,0,-2,0,1,0,0,0) | 13- | 78/49 |
68: (1,-2,1,-1,1,0,0,0,0) | 11- | 110/63 |
69: (-4,1,1,-1,1,0,0,0,0) | 11- | 165/112 |
70: (-3,0,0,0,1,0,0,0,0) | 11- | 11/8 | *undecimal half-augmented fourth*
71: (-1,0,-1,0,1,0,0,0,0) | 11- | 11/10 | *large undecimal neutral second*
72: (3,0,0,-2,1,0,0,0,0) | 11- | 88/49 |
73: (0,0,1,-2,0,1,0,0,0) | 13- | 65/49 |

74: (5,1,0,-2,0,0,0,0,0) | 7- | 96/49 |
75: (-1,1,1,-2,0,1,0,0,0) | 13- | 195/98 |
76: (2,0,0,-2,0,1,0,0,0) | 13- | 52/49 |
77: (2,0,-1,-1,0,1,0,0,0) | 13- | 52/35 |
78: (0,1,0,-2,0,0,1,0,0) | 17- | 51/49 |
79: (0,1,-1,-1,0,0,1,0,0) | 17- | 51/35 |
80: (-1,2,0,-2,0,1,0,0,0) | 13- | 117/98 |
81: (-1,2,-1,-1,0,1,0,0,0) | 13- | 117/70 |
82: (0,4,0,-2,0,0,0,0,0) | 7- | 81/49 |
83: (0,1,0,-2,0,0,0,1,0) | 19- | 57/49 |
84: (0,1,-1,-1,0,0,0,1,0) | 19- | 57/35 |
85: (-1,4,-1,-1,0,0,0,0,0) | 7- | 81/70 |
86: (-3,4,0,-1,0,0,0,0,0) | 7- | 81/56 |
87: (-3,1,0,-1,0,0,0,1,0) | 19- | 57/56 |
88: (-4,3,1,-1,0,0,0,0,0) | 7- | 135/112 |
89: (1,0,-1,-1,0,0,0,1,0) | 19- | 38/35 |
90: (3,1,1,-1,-1,0,0,0,0) | 11- | 120/77 |
91: (0,1,0,-2,0,0,0,0,1) | 23- | 69/49 |
92: (0,1,-1,-1,0,0,0,0,1) | 23- | 69/35 |
93: (-3,1,0,-1,0,0,0,0,1) | 23- | 69/56 |
94: (2,1,0,0,-1,0,0,0,0) | 11- | 12/11 | *small undecimal neutral second*
95: (1,0,-1,-1,0,0,0,0,1) | 23- | 46/35 |
96: (0,0,-1,1,0,0,0,0,0) | 7- | 7/5 | *septimal diminished fifth*
97: (-2,1,-1,0,1,0,0,0,0) | 11- | 33/20 |
98: (1,2,0,0,-1,0,0,0,0) | 11- | 18/11 | *undecimal neutral sixth*
99: (-2,3,-1,0,0,0,0,0,0) | 5- | 27/20 | *ptolemaic wide fourth*
100: (-1,0,-1,0,0,1,0,0,0) | 13- | 13/10 | *tridecimal half-diminished fourth*
101: (-3,0,0,0,0,1,0,0,0) | 13- | 13/8 | *tridecimal neutral sixth*
102: (-4,1,1,-1,0,1,0,0,0) | 13- | 195/112 |
103: (-4,2,0,-1,0,1,0,0,0) | 13- | 117/112 |
104: (1,1,0,-1,-1,1,0,0,0) | 13- | 78/77 |
105: (-2,1,-1,0,0,1,0,0,0) | 13- | 39/20 |
106: (-2,-1,0,0,0,1,0,0,0) | 13- | 13/12 | *large tridecimal neutral second*
107: (-4,3,0,0,0,0,0,0,0) | 3- | 27/16 | *pythagorean major sixth*
108: (-2,1,-1,1,0,0,0,0,0) | 7- | 21/20 | *minor diatonic semitone*
109: (1,2,0,0,0,-1,0,0,0) | 13- | 18/13 | *tridecimal augmented fourth*
110: (-2,2,2,-2,0,0,0,0,0) | 7- | 225/196 |
111: (0,3,1,-1,-1,0,0,0,0) | 11- | 135/77 |
112: (2,3,0,-1,-1,0,0,0,0) | 11- | 108/77 |
113: (0,2,0,-1,-1,1,0,0,0) | 13- | 117/77 |
114: (0,-2,1,-1,0,1,0,0,0) | 13- | 65/63 |
115: (2,2,1,-1,0,-1,0,0,0) | 13- | 180/91 |
116: (3,1,1,-1,0,-1,0,0,0) | 13- | 120/91 |

117: (1,1,2,-1,-1,0,0,0,0) | 11- | 150/77 |
118: (-1,1,1,-1,-1,1,0,0,0) | 13- | 195/154 |
119: (2,0,1,0,-1,0,0,0,0) | 11- | 20/11 | *small undecimal neutral seventh*
120: (4,2,0,-1,-1,0,0,0,0) | 11- | 144/77 |
121: (0,3,1,-1,0,-1,0,0,0) | 13- | 135/91 |
122: (-5,2,1,0,0,0,0,0,0) | 5- | 45/32 | *ptolemaic tritone*
123: (-2,2,1,0,-1,0,0,0,0) | 11- | 45/44 | *large chromatic undecimal sixthtone*
124: (-1,3,0,0,-1,0,0,0,0) | 11- | 27/22 |
125: (0,4,0,-1,-1,0,0,0,0) | 11- | 81/77 |
126: (-1,2,2,-1,-1,0,0,0,0) | 11- | 225/154 |
127: (-1,2,1,0,0,-1,0,0,0) | 13- | 45/26 |
128: (2,3,0,-1,0,-1,0,0,0) | 13- | 108/91 |
129: (5,1,0,-1,-1,0,0,0,0) | 11- | 96/77 |
130: (4,0,1,-1,-1,0,0,0,0) | 11- | 80/77 |
131: (1,0,1,-1,-1,1,0,0,0) | 13- | 130/77 |
132: (2,2,1,-1,0,0,-1,0,0) | 17- | 180/119 |
133: (4,-2,1,-1,0,0,0,0,0) | 7- | 80/63 |
134: (1,1,2,-1,0,-1,0,0,0) | 13- | 150/91 |
135: (1,1,1,0,0,0,-1,0,0) | 17- | 30/17 |
136: (3,1,1,-1,0,0,-1,0,0) | 17- | 120/119 |
137: (1,2,0,0,0,0,-1,0,0) | 17- | 18/17 | *Galileo's 99¢ semitone*
138: (3,1,0,0,0,-1,0,0,0) | 13- | 24/13 | *tridecimal neutral seventh*
139: (2,2,1,-1,0,0,0,-1,0) | 19- | 180/133 |
140: (1,1,1,0,0,0,0,-1,0) | 19- | 30/19 |
141: (2,0,1,0,0,-1,0,0,0) | 13- | 20/13 | *tridecimal half-augmented fifth*
142: (2,0,2,-1,-1,0,0,0,0) | 11- | 100/77 |
143: (0,1,1,-1,1,-1,0,0,0) | 13- | 165/91 |
144: (4,2,0,-1,0,-1,0,0,0) | 13- | 144/91 |
145: (0,2,0,-1,1,-1,0,0,0) | 13- | 99/91 |
146: (2,1,0,-1,1,-1,0,0,0) | 13- | 132/91 |
147: (0,3,1,-1,0,0,-1,0,0) | 17- | 135/119 |
148: (6,0,-1,-1,0,0,0,0,0) | 7- | 64/35 | *septimal neutral seventh*
149: (5,1,0,-1,0,-1,0,0,0) | 13- | 96/91 |
150: (2,-2,2,-1,0,0,0,0,0) | 7- | 100/63 |
151: (2,2,1,-1,0,0,0,0,-1) | 23- | 180/161 |
152: (-1,2,2,-1,0,-1,0,0,0) | 13- | 225/182 |
153: (-1,3,0,0,0,-1,0,0,0) | 13- | 27/26 | *tridecimal thirdtone*
154: (1,4,0,-1,0,-1,0,0,0) | 13- | 162/91 |
155: (1,0,1,-1,1,-1,0,0,0) | 13- | 110/91 |
156: (2,0,2,-1,0,-1,0,0,0) | 13- | 100/91 |
157: (1,1,2,-1,0,0,-1,0,0) | 17- | 150/119 |
158: (1,1,1,0,0,0,0,0,-1) | 23- | 30/23 |
159: (0,3,1,-1,0,0,0,-1,0) | 19- | 135/133 |

160: (-1,2,1,0,0,0,-1,0,0) | 17- | 45/34 |
161: (-3,0,1,-1,0,0,1,0,0) | 17- | 85/56 |
162: (-3,0,1,-1,0,0,0,1,0) | 19- | 95/56 |
163: (-5,2,2,-1,0,0,0,0,0) | 7- | 225/224 |
164: (1,1,2,-1,0,0,0,-1,0) | 19- | 150/133 |
165: (2,0,1,0,0,0,-1,0,0) | 17- | 20/17 | *17-limit augmented second*
166: (3,1,0,0,0,0,-1,0,0) | 17- | 24/17 | *17-limit tritone*
167: (0,2,2,-1,0,0,-1,0,0) | 17- | 225/119 |
168: (0,1,1,-1,1,0,-1,0,0) | 17- | 165/119 |
169: (4,2,0,-1,0,0,-1,0,0) | 17- | 144/119 |
170: (2,2,-2,0,0,0,0,0,0) | 5- | 36/25 | *Rameau's false fifth*
171: (-1,2,1,0,0,0,0,-1,0) | 19- | 45/38 |
172: (2,2,0,0,0,0,0,-1,0) | 19- | 36/19 |
173: (4,1,1,-1,0,0,0,-1,0) | 19- | 240/133 |
174: (0,2,1,0,0,0,0,0,-1) | 23- | 45/23 |
175: (-4,0,2,0,0,0,0,0,0) | 5- | 25/16 | *ptolemaic augmented fifth*
176: (2,0,1,0,0,0,0,-1,0) | 19- | 20/19 | *small 19-limit semitone*
177: (3,1,0,0,0,0,0,-1,0) | 19- | 24/19 |
178: (4,0,0,0,-1,0,0,0,0) | 11- | 16/11 | *undecimal half-diminished fifth*
179: (-4,1,0,1,0,0,0,0,0) | 7- | 21/16 | *septimal narrow fourth*
180: (0,1,0,1,-1,0,0,0,0) | 11- | 21/11 | *undecimal diminished octave*
181: (0,3,0,0,0,0,-1,0,0) | 17- | 27/17 |
182: (4,-2,0,0,0,0,0,0,0) | 3- | 16/9 | *pythagorean minor seventh*
183: (4,2,0,-1,0,0,0,-1,0) | 19- | 144/133 |
184: (4,-1,-1,0,0,0,0,0,0) | 5- | 16/15 | *major diatonic semitone*
185: (2,0,-1,-1,0,0,1,0,0) | 17- | 68/35 |
186: (4,0,0,0,0,-1,0,0,0) | 13- | 16/13 | *major tridecimal neutral third*
187: (2,2,0,0,0,0,0,0,-1) | 23- | 36/23 |
188: (6,0,0,-2,0,0,0,0,0) | 7- | 64/49 |
189: (3,3,0,-1,0,0,-1,0,0) | 17- | 216/119 |
190: (1,1,0,-1,-1,0,1,0,0) | 17- | 102/77 |

Harmonic Space constrained by 1/1 ≤ P ≤ 2/1

Harmonic Space Co-ordinates | Prime Limit | Ratio | Interval
dimensions: 2,3,5,7,11,13,17,19,23

1: (0,0,0,0,0,0,0,0,0) | 2- | 1/1 | *unison*
2: (1,0,0,0,0,0,0,0,0) | 2- | 2/1 | *octave*

3: (-1,1,0,0,0,0,0,0,0) | 3- | 3/2 | *fifth*
4: (2,-1,0,0,0,0,0,0,0) | 3- | 4/3 | *fourth*

5: (0,-1,1,0,0,0,0,0,0) | 5- | 5/3 | *ptolemaic major sixth*
6: (-2,0,1,0,0,0,0,0,0) | 5- | 5/4 | *ptolemaic major third*
7: (1,-2,1,0,0,0,0,0,0) | 5- | 10/9 | *minor wholetone (subdominant wholetone)*

8: (1,0,1,-1,0,0,0,0,0) | 7- | 10/7 | *septimal tritone*
9: (-3,1,1,0,0,0,0,0,0) | 5- | 15/8 | *ptolemaic major seventh*
10: (-1,1,1,-1,0,0,0,0,0) | 7- | 15/14 | *major chromatic semitone*
11: (1,1,-1,0,0,0,0,0,0) | 5- | 6/5 | *ptolemaic minor third*
12: (2,1,0,-1,0,0,0,0,0) | 7- | 12/7 | *septimal major sixth*
13: (3,0,0,-1,0,0,0,0,0) | 7- | 8/7 | *septimal wholetone*
14: (0,2,0,-1,0,0,0,0,0) | 7- | 9/7 | *septimal major third*
15: (3,0,-1,0,0,0,0,0,0) | 5- | 8/5 | *ptolemaic minor sixth*
16: (0,2,-1,0,0,0,0,0,0) | 5- | 9/5 | *ptolemaic minor seventh*
17: (-3,2,0,0,0,0,0,0,0) | 3- | 9/8 | *major wholetone (dominant wholetone)*
18: (-2,0,0,1,0,0,0,0,0) | 7- | 7/4 | *natural seventh*
19: (-1,-1,0,1,0,0,0,0,0) | 7- | 7/6 | *septimal minor third*
20: (0,0,-1,1,0,0,0,0,0) | 7- | 7/5 | *septimal diminished fifth*
21: (1,-2,0,1,0,0,0,0,0) | 7- | 14/9 | *septimal minor sixth*
22: (-2,1,-1,1,0,0,0,0,0) | 7- | 21/20 | *minor diatonic semitone*
23: (-4,1,0,1,0,0,0,0,0) | 7- | 21/16 | *septimal narrow fourth*
24: (2,-1,-1,1,0,0,0,0,0) | 7- | 28/15 | *septimal diminished octave*

25: (2,1,0,0,-1,0,0,0,0) | 11- | 12/11 | *small undecimal neutral second*
26: (4,-1,-1,0,0,0,0,0,0) | 5- | 16/15 | *major diatonic semitone*
27: (4,-2,0,0,0,0,0,0,0) | 3- | 16/9 | *pythagorean minor seventh*
28: (-1,-1,0,0,1,0,0,0,0) | 11- | 11/6 | *large undecimal neutral seventh*
29: (-1,0,-1,0,1,0,0,0,0) | 11- | 11/10 | *large undecimal neutral second*
30: (-3,0,0,0,1,0,0,0,0) | 11- | 11/8 | *undecimal half-augmented fourth*
31: (1,-1,-1,0,1,0,0,0,0) | 11- | 22/15 | *undecimal diminished fifth*
32: (0,-2,0,0,1,0,0,0,0) | 11- | 11/9 | *undecimal neutral third*
33: (0,0,0,-1,1,0,0,0,0) | 11- | 11/7 | *undecimal augmented fifth*
34: (-2,1,-1,0,1,0,0,0,0) | 11- | 33/20 |

35: (1,-1,0,-1,1,0,0,0,0) | 11- | 22/21 | *undecimal semitone*
36: (-2,1,0,-1,1,0,0,0,0) | 11- | 33/28 |
37: (1,1,-1,-1,1,0,0,0,0) | 11- | 66/35 |
38: (2,0,-1,-1,1,0,0,0,0) | 11- | 44/35 |
39: (2,2,-1,-1,0,0,0,0,0) | 7- | 36/35 | *septimal quartertone*
40: (4,1,-1,-1,0,0,0,0,0) | 7- | 48/35 | *septimal half-augmented fourth*
41: (-1,3,0,-1,0,0,0,0,0) | 7- | 27/14 | *small septimal major seventh*
42: (-2,3,-1,0,0,0,0,0,0) | 5- | 27/20 | *ptolemaic wide fourth*
43: (-1,0,-1,0,0,1,0,0,0) | 13- | 13/10 | *tridecimal half-diminished fourth*
44: (0,0,0,-1,0,1,0,0,0) | 13- | 13/7 | *tridecimal major seventh*
45: (1,3,-1,-1,0,0,0,0,0) | 7- | 54/35 |
46: (1,2,0,0,-1,0,0,0,0) | 11- | 18/11 | *undecimal neutral sixth*
47: (-2,2,1,-1,0,0,0,0,0) | 7- | 45/28 |
48: (3,-1,1,-1,0,0,0,0,0) | 7- | 40/21 | *large septimal major seventh*
49: (5,-1,0,-1,0,0,0,0,0) | 7- | 32/21 | *septimal wide fifth*
50: (2,2,-2,0,0,0,0,0,0) | 5- | 36/25 | *Rameau's false fifth*

51: (1,-1,-1,0,0,1,0,0,0) | 13- | 26/15 |
52: (1,-1,0,-1,0,1,0,0,0) | 13- | 26/21 |
53: (-2,-1,0,0,0,1,0,0,0) | 13- | 13/12 | *large tridecimal neutral second*
54: (-3,0,0,0,0,1,0,0,0) | 13- | 13/8 | *tridecimal neutral sixth*
55: (-2,1,-1,0,0,1,0,0,0) | 13- | 39/20 |
56: (0,1,-1,-1,0,1,0,0,0) | 13- | 39/35 |
57: (-2,1,0,-1,0,1,0,0,0) | 13- | 39/28 |
58: (2,0,-1,-1,0,1,0,0,0) | 13- | 52/35 |
59: (0,-2,0,0,0,1,0,0,0) | 13- | 13/9 | *tridecimal diminished fifth*

60: (-1,0,-1,0,0,0,1,0,0) | 17- | 17/10 | *17-limit diminished seventh*
61: (-1,0,0,-1,0,0,1,0,0) | 17- | 17/14 | *17-limit minor third*
62: (-1,0,2,-1,0,0,0,0,0) | 7- | 25/14 | *septimal augmented sixth*
63: (-1,2,-1,-1,1,0,0,0,0) | 11- | 99/70 |

64: (-1,0,-1,0,0,0,0,1,0) | 19- | 19/10 | *19-limit diminished octave*
65: (-1,0,0,-1,0,0,0,1,0) | 19- | 19/14 | *19-limit wide fourth*
66: (-1,2,-1,-1,0,1,0,0,0) | 13- | 117/70 |
67: (0,-1,-1,0,0,0,1,0,0) | 17- | 17/15 | *17-limit diminished third*
68: (0,1,-1,-1,0,0,1,0,0) | 17- | 51/35 |
69: (1,-1,0,-1,0,0,1,0,0) | 17- | 34/21 |
70: (-2,-1,0,0,0,0,1,0,0) | 17- | 17/12 | *17-limit diminished fifth*
71: (-2,1,0,-1,0,0,1,0,0) | 17- | 51/28 |
72: (2,0,-1,-1,0,0,1,0,0) | 17- | 68/35 |
73: (1,0,-1,-1,0,0,0,1,0) | 19- | 38/35 |
74: (0,1,-1,-1,0,0,0,1,0) | 19- | 57/35 |

75: (0,-1,-1,0,0,0,0,1,0) | 19- | 19/15 |
76: (1,-1,0,-1,0,0,0,1,0) | 19- | 38/21 |
77: (-2,-1,0,0,0,0,0,1,0) | 19- | 19/12 | *19-limit minor sixth*
78: (0,-1,2,-1,0,0,0,0,0) | 7- | 25/21 | *septimal augmented second*

79: (-1,0,0,-1,0,0,0,0,1) | 23- | 23/14 |
80: (-2,0,1,-1,1,0,0,0,0) | 11- | 55/28 |
81: (-1,-1,1,-1,1,0,0,0,0) | 11- | 55/42 |
82: (-1,-1,1,-1,0,1,0,0,0) | 13- | 65/42 |
83: (0,-1,0,-1,0,0,0,0,1) | 23- | 23/21 |
84: (1,0,-1,-1,0,0,0,0,1) | 23- | 46/35 |
85: (6,0,-1,-1,0,0,0,0,0) | 7- | 64/35 | *septimal neutral seventh*
86: (0,1,-1,-1,0,0,0,0,1) | 23- | 69/35 |
87: (0,-1,-1,0,0,0,0,0,1) | 23- | 23/15 |
88: (-2,0,-1,0,0,0,0,0,1) | 23- | 23/20 |
89: (-2,-1,0,0,0,0,0,0,1) | 23- | 23/12 | *23-limit major seventh*
90: (0,-2,0,0,0,0,1,0,0) | 17- | 17/9 | *17-limit diminished octave*
91: (-1,4,-1,-1,0,0,0,0,0) | 7- | 81/70 |
92: (0,1,1,0,-1,0,0,0,0) | 11- | 15/11 | *undecimal augmented fourth*
93: (0,1,-2,0,1,0,0,0,0) | 11- | 33/25 |
94: (1,0,-2,0,0,1,0,0,0) | 13- | 26/25 | *tridecimal semitone*
95: (0,3,-2,0,0,0,0,0,0) | 5- | 27/25 | *comma-augmented semitone*
96: (0,1,-2,0,0,1,0,0,0) | 13- | 39/25 |
97: (4,1,-2,0,0,0,0,0,0) | 5- | 48/25 | *large diminished octave*
98: (1,1,-2,1,0,0,0,0,0) | 7- | 42/25 |
99: (1,2,0,0,0,-1,0,0,0) | 13- | 18/13 | *tridecimal augmented fourth*
100: (2,0,-2,1,0,0,0,0,0) | 7- | 28/25 | *septimal diminished third*
101: (2,0,-2,0,1,0,0,0,0) | 11- | 44/25 |
102: (-4,3,0,0,0,0,0,0,0) | 3- | 27/16 | *pythagorean major sixth*
103: (-3,1,-1,0,0,0,1,0,0) | 17- | 51/40 |
104: (-3,2,-1,1,0,0,0,0,0) | 7- | 63/40 | *narrow sept. minor sixth*
105: (3,1,0,0,0,-1,0,0,0) | 13- | 24/13 | *tridecimal neutral seventh*
106: (0,1,1,0,0,-1,0,0,0) | 13- | 15/13 | *tridecimal augmented second*
107: (5,0,-2,0,0,0,0,0,0) | 5- | 32/25 | *ptolemaic diminished fourth*
108: (1,0,-2,0,0,0,1,0,0) | 17- | 34/25 |
109: (1,2,0,0,0,0,-1,0,0) | 17- | 18/17 | *Galileo's 99¢ semitone*
110: (4,0,0,0,-1,0,0,0,0) | 11- | 16/11 | *undecimal half-diminished fifth*
111: (2,0,1,0,-1,0,0,0,0) | 11- | 20/11 | *small undecimal neutral seventh*
112: (1,0,0,1,-1,0,0,0,0) | 11- | 14/11 | *undecimal diminished fourth*
113: (0,1,0,1,-1,0,0,0,0) | 11- | 21/11 | *undecimal diminished octave*
114: (0,0,0,0,-1,1,0,0,0) | 13- | 13/11 | *tridecimal minor third*
115: (2,-2,-1,0,0,1,0,0,0) | 13- | 52/45 |
116: (3,1,0,0,0,0,-1,0,0) | 17- | 24/17 | *17-limit tritone*

117: (4,0,0,0,0,-1,0,0,0) | 13- | 16/13 | *major tridecimal neutral third*
118: (2,0,1,0,0,-1,0,0,0) | 13- | 20/13 | *tridecimal half-augmented fifth*
119: (1,0,0,1,0,-1,0,0,0) | 13- | 14/13 | *small tridecimal neutral second*
120: (0,1,0,1,0,-1,0,0,0) | 13- | 21/13 |
121: (1,0,0,0,1,-1,0,0,0) | 13- | 22/13 |
122: (3,1,0,0,0,0,0,-1,0) | 19- | 24/19 |
123: (1,1,1,0,0,0,-1,0,0) | 17- | 30/17 |
124: (-4,0,0,0,0,0,0,1,0,0) | 17- | 17/16 | *large 17-limit semitone*
125: (3,-2,-1,1,0,0,0,0,0) | 7- | 56/45 |
126: (2,0,1,0,0,0,-1,0,0) | 17- | 20/17 | *17-limit augmented second*
127: (1,0,-2,0,0,0,0,1,0) | 19- | 38/25 |
128: (-1,-2,0,0,0,0,0,1,0) | 19- | 19/18 | *large 19-limit semitone*
129: (-3,1,-1,0,0,0,0,1,0) | 19- | 57/40 |
130: (-1,2,-2,1,0,0,0,0,0) | 7- | 63/50 |
131: (-1,1,-2,0,0,0,1,0,0) | 17- | 51/50 |
132: (-1,-2,1,1,0,0,0,0,0) | 7- | 35/18 | *septimal half-diminished octave*
133: (-3,-1,1,1,0,0,0,0,0) | 7- | 35/24 | *septimal half-diminished fifth*
134: (-1,-2,2,0,0,0,0,0,0) | 5- | 25/18 | *Rameau's tritone*
135: (-3,-1,2,0,0,0,0,0,0) | 5- | 25/24 | *minor chrom. semitone*
136: (-1,-1,-1,2,0,0,0,0,0) | 7- | 49/30 |
137: (0,1,0,1,0,0,-1,0,0) | 17- | 21/17 |
138: (-1,3,0,0,-1,0,0,0,0) | 11- | 27/22 |
139: (1,1,1,0,0,0,0,-1,0) | 19- | 30/19 |
140: (-4,0,0,0,0,0,0,0,1,0) | 19- | 19/16 | *19-limit minor third*
141: (-4,0,2,0,0,0,0,0,0,0) | 5- | 25/16 | *ptolemaic augmented fifth*
142: (2,2,0,0,0,0,0,0,-1,0) | 19- | 36/19 |
143: (2,0,1,0,0,0,0,0,-1,0) | 19- | 20/19 | *small 19-limit semitone*
144: (3,1,0,0,0,0,0,0,0,-1) | 23- | 24/23 | *23-limit semitone*
145: (0,1,0,1,0,0,0,-1,0) | 19- | 21/19 |
146: (2,-3,0,1,0,0,0,0,0) | 7- | 28/27 | *septimal thirdtone*
147: (-1,-2,0,0,0,0,0,0,0,1) | 23- | 23/18 |
148: (0,0,0,0,-1,0,1,0,0) | 17- | 17/11 |
149: (-1,1,0,0,-1,1,0,0,0) | 13- | 39/22 |
150: (3,2,-1,0,-1,0,0,0,0) | 11- | 72/55 |
151: (2,1,-1,1,-1,0,0,0,0) | 11- | 84/55 |
152: (-1,3,0,0,0,-1,0,0,0) | 13- | 27/26 | *tridecimal thirdtone*
153: (-1,2,1,0,0,-1,0,0,0) | 13- | 45/26 |
154: (1,1,1,0,0,0,0,0,-1) | 23- | 30/23 |
155: (-1,1,0,0,1,-1,0,0,0) | 13- | 33/26 |
156: (-5,2,1,0,0,0,0,0,0) | 5- | 45/32 | *ptolemaic tritone*
157: (2,2,0,0,0,0,0,0,-1) | 23- | 36/23 |
158: (2,1,1,-2,0,0,0,0,0) | 7- | 60/49 |
159: (-5,1,0,0,1,0,0,0,0) | 11- | 33/32 | *undecimal quartertone*

160: (-2,2,1,0,-1,0,0,0,0) | 11- | 45/44 | *large chromatic undecimal sixthtone*
161: (-3,1,-1,0,0,0,0,0,1) | 23- | 69/40 |
162: (3,2,-1,0,0,-1,0,0,0) | 13- | 72/65 |
163: (0,3,0,0,0,0,-1,0,0) | 17- | 27/17 |
164: (2,0,0,1,0,0,-1,0,0) | 17- | 28/17 |
165: (-1,0,1,1,-1,0,0,0,0) | 11- | 35/22 |
166: (3,-1,1,0,-1,0,0,0,0) | 11- | 40/33 |
167: (0,0,0,0,-1,0,0,1,0) | 19- | 19/11 |
168: (3,-3,1,0,0,0,0,0,0) | 5- | 40/27 | *ptolemaic narrow fifth*
169: (-1,0,2,0,-1,0,0,0,0) | 11- | 25/22 |
170: (-1,2,1,0,0,0,-1,0,0) | 17- | 45/34 |
171: (1,0,0,0,1,0,-1,0,0) | 17- | 22/17 |
172: (0,1,0,0,1,0,-1,0,0) | 17- | 33/17 |
173: (-4,0,0,0,0,0,0,0,1) | 23- | 23/16 | *23-limit tritone*
174: (-5,1,0,0,0,1,0,0,0) | 13- | 39/32 | *minor tridecimal neutral third*
175: (0,0,0,0,0,-1,1,0,0) | 17- | 17/13 |
176: (0,3,0,0,0,0,0,-1,0) | 19- | 27/19 |
177: (2,0,0,1,0,0,0,-1,0) | 19- | 28/19 |
178: (-5,2,0,1,0,0,0,0,0) | 7- | 63/32 | *septimal narrow octave*
179: (0,2,-1,1,-1,0,0,0,0) | 11- | 63/55 |
180: (-2,2,0,1,-1,0,0,0,0) | 11- | 63/44 |
181: (1,1,0,1,0,0,0,0,-1) | 23- | 42/23 |
182: (2,1,-1,1,0,-1,0,0,0) | 13- | 84/65 |
183: (-1,0,1,1,0,-1,0,0,0) | 13- | 35/26 |
184: (-5,0,1,1,0,0,0,0,0) | 7- | 35/32 | *septimal neutral second*
185: (0,0,2,0,0,-1,0,0,0) | 13- | 25/13 |
186: (-1,2,1,0,0,0,0,-1,0) | 19- | 45/38 |
187: (-3,1,2,-1,0,0,0,0,0) | 7- | 75/56 |
188: (0,-1,1,1,-1,0,0,0,0) | 11- | 35/33 |
189: (3,-1,1,0,0,-1,0,0,0) | 13- | 40/39 | *tridecimal quartertone*
190: (3,0,-1,1,-1,0,0,0,0) | 11- | 56/55 | *diatonic undecimal sixthtone*

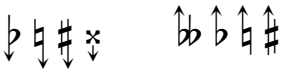
ACCIDENTALS

EXTENDED HELMHOLTZ-ELLIS JI PITCH NOTATION

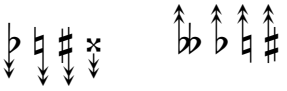
for Just Intonation
designed by Marc Sabat and Wolfgang von Schweinitz

The exact intonation of each pitch may be written out by means of the following harmonically-defined signs:

Pythagorean series of fifths – the open strings
(… c g d a e …)



lowers / raises by a syntonic comma
81 : 80 = ***circa 21.5 cents***



lowers / raises by two syntonic commas
circa 43 cents

lowers / raises by a septimal comma
64 : 63 = ***circa 27.3 cents***

lowers / raises by two septimal commas
circa 54.5 cents

raises / lowers by an 11-limit undecimal quarter-tone
33 : 32 = ***circa 53.3 cents***

lowers / raises by a 13-limit tridecimal third-tone
27 : 26 = ***circa 65.3 cents***

lowers / raises by a 17-limit schisma
256 : 255 = ***circa 6.8 cents***

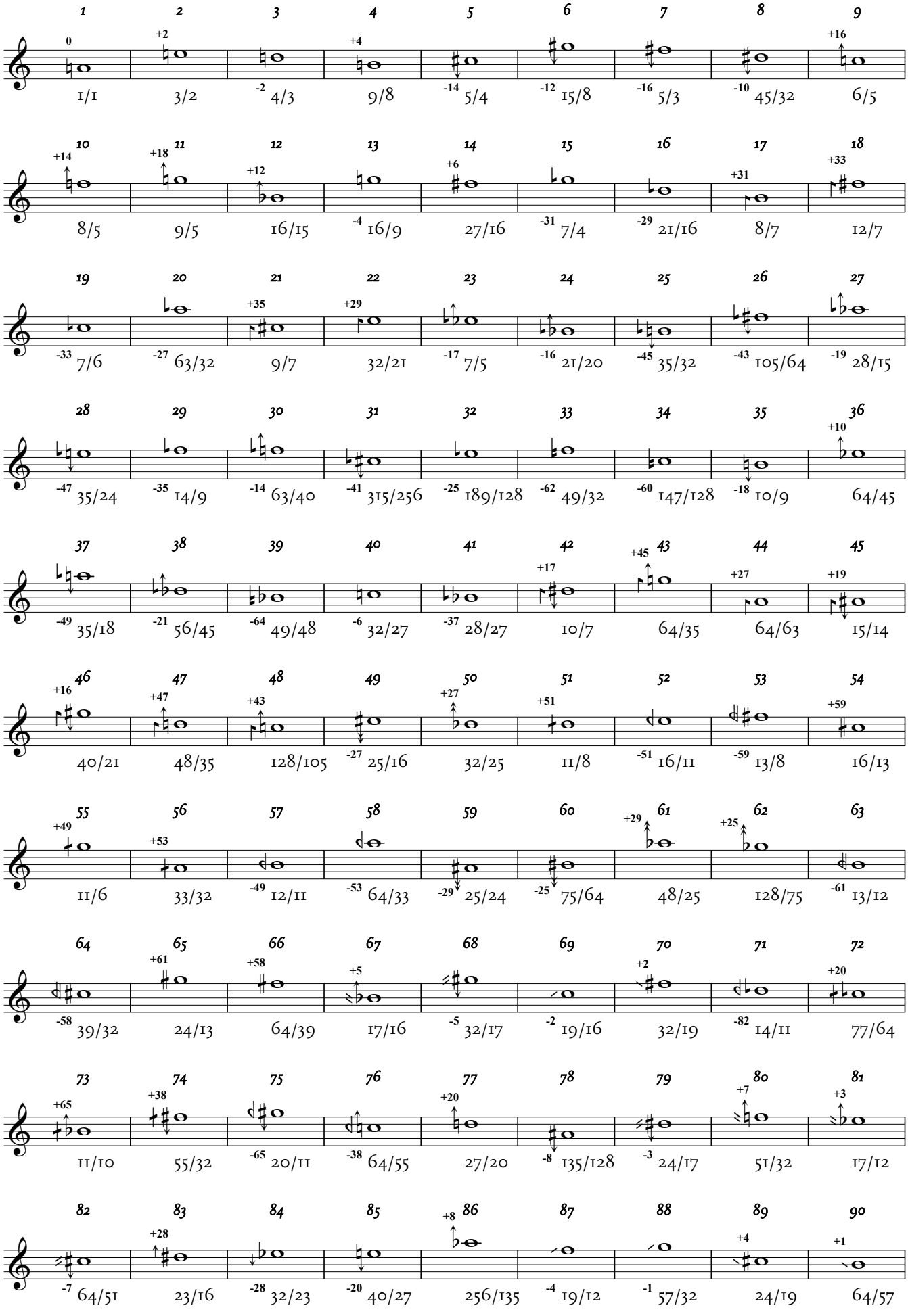
raises / lowerss by a 19-limit schisma
513 : 512 = ***circa 3.4 cents***

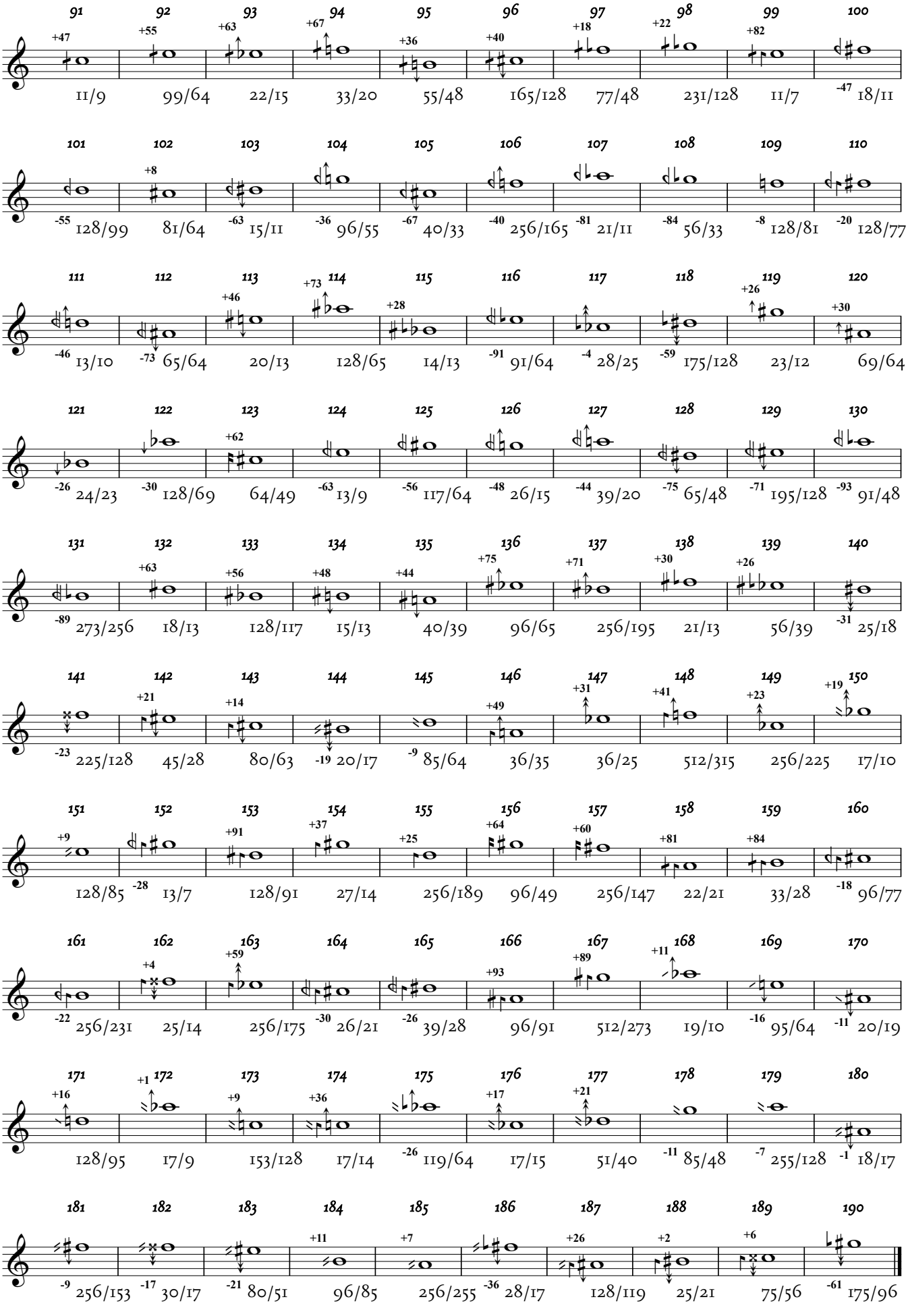
raises / lowers by a 23-limit comma
736 : 729 = ***circa 16.5 cents***

In addition to the harmonic definition of a pitch by means of its accidentals, it is also possible to indicate its absolute pitch-height as a cents-deviation from the respectively indicated chromatic pitch in the 12-tone system of Equal Temperament.

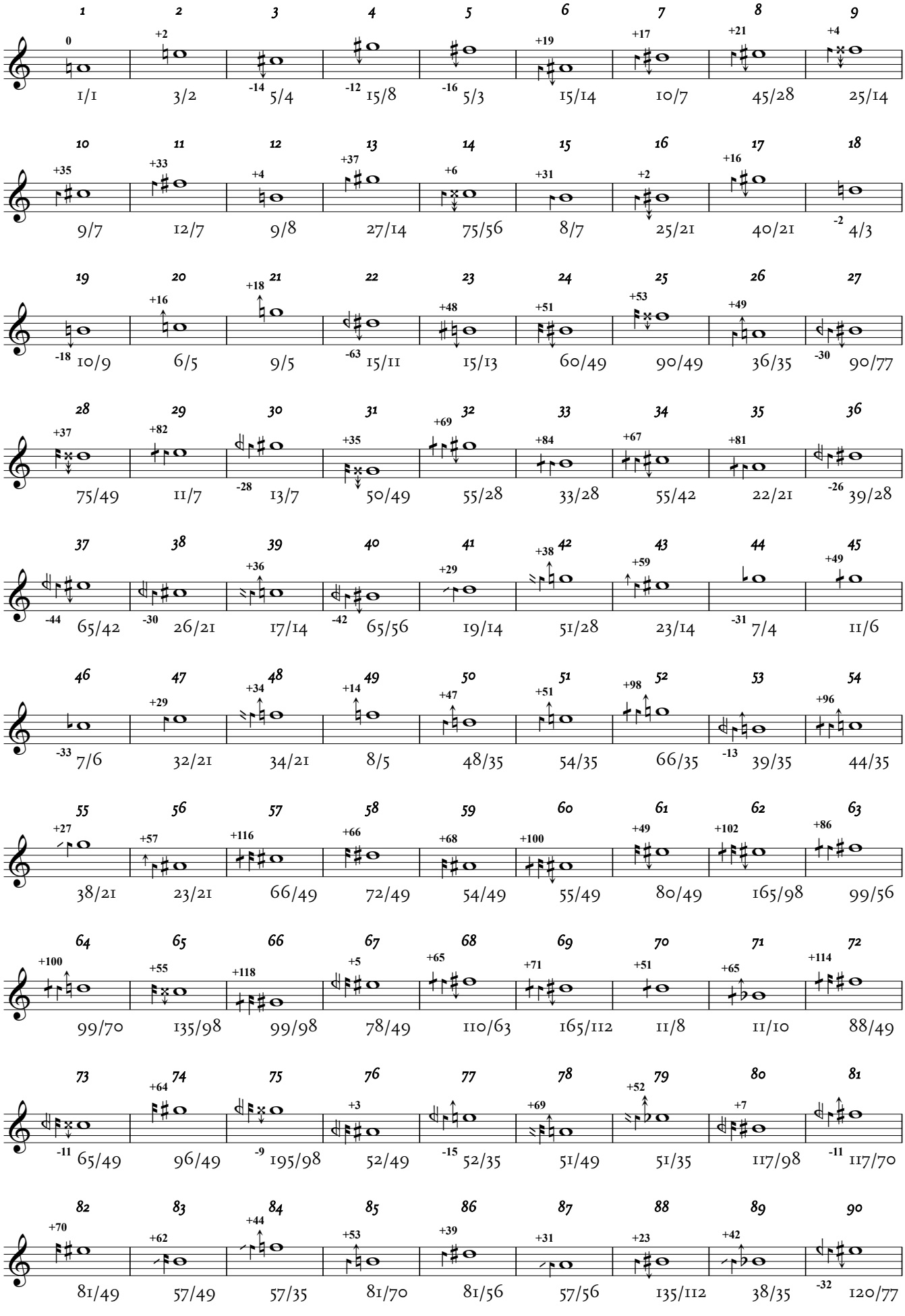
The attached arrows for alteration by a syntonic comma are transcriptions of the notation that Hermann von Helmholtz used in his book "Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik" (1863). The annotated English translation "On the Sensations of Tone as a Physiological Basis for the Theory of Music" (1875/1885) is by Alexander J. Ellis, who refined the definition of pitch within the 12-tone system of Equal Temperament by introducing a division of the octave into 1200 cents. The sign for a septimal comma was devised by Giuseppe Tartini (1692-1770) – the composer, violinist and researcher who first studied the production of difference tones by means of double stops.

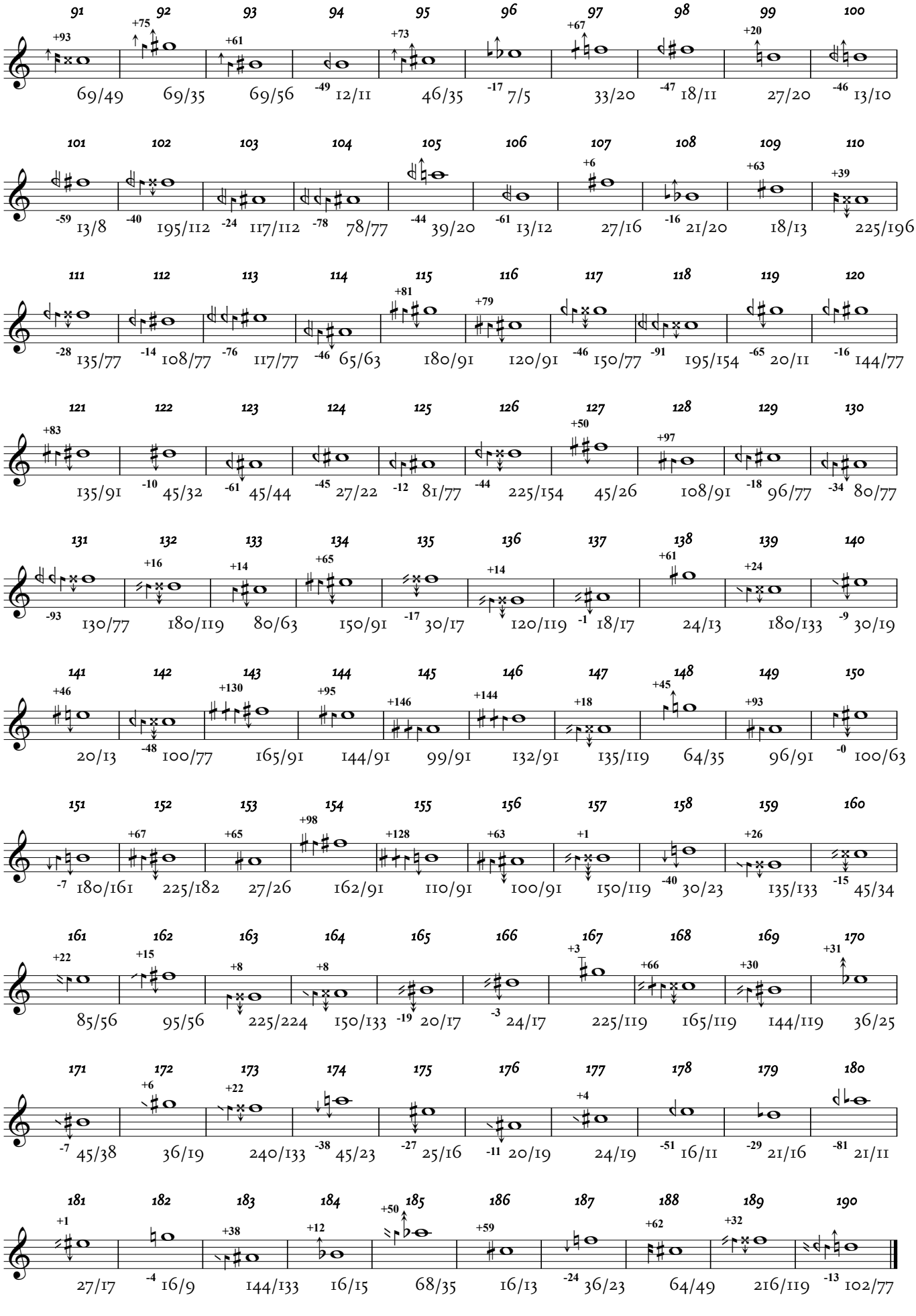
190 points in 23-limit pitch-class projection space



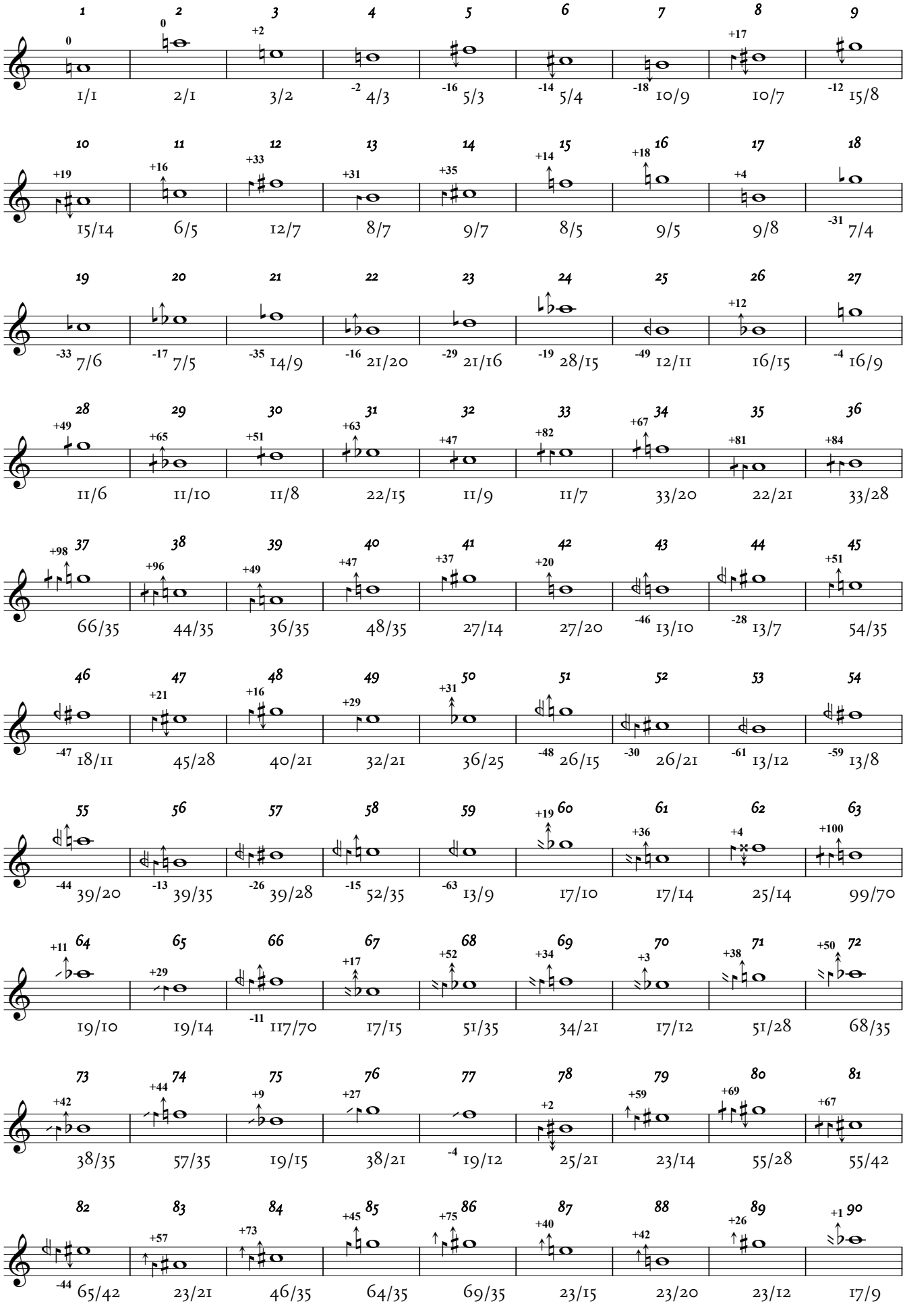

91 92 93 94 95 96 97 98 99 100
+47 +55 +63 +67 +36 +40 +18 +22 +82
11/9 99/64 22/15 33/20 55/48 165/128 77/48 231/128 11/7 -47 18/11
101 102 103 104 105 106 107 108 109 110
+8
-55 128/99 81/64 -63 15/11 -36 96/55 -67 40/33 -40 256/165 -81 21/11 -84 56/33 -8 128/81 -20 128/77
111 112 113 114 115 116 117 118 119 120
+46 +73 +28 +26 +30
-46 13/10 -73 65/64 20/13 128/65 14/13 -91 91/64 -4 28/25 -59 175/128 23/12 69/64
121 122 123 124 125 126 127 128 129 130
+62
-26 24/23 -30 128/69 64/49 -63 13/9 -56 117/64 -48 26/15 -44 39/20 -75 65/48 -71 195/128 -93 91/48
131 132 133 134 135 136 137 138 139 140
+63 +56 +48 +44 +75 +71 +30 +26
-89 273/256 18/13 128/117 15/13 40/39 96/65 256/195 21/13 56/39 -31 25/18
141 142 143 144 145 146 147 148 149 150
+21 +14 +49 +31 +41 +23 +19
-23 225/128 45/28 80/63 -19 20/17 -9 85/64 36/35 36/25 512/315 256/225 17/10
151 152 153 154 155 156 157 158 159 160
+9 +91 +37 +25 +64 +60 +81 +84
128/85 -28 13/7 128/91 27/14 256/189 96/49 256/147 22/21 33/28 -18 96/77
161 162 163 164 165 166 167 168 169 170
+4 +59 +93 +89 +11
-22 256/231 25/14 256/175 -30 26/21 -26 39/28 96/91 512/273 19/10 -16 95/64 -11 20/19
171 172 173 174 175 176 177 178 179 180
+16 +1 +9 +36 +17 +21
128/95 17/9 153/128 17/14 -26 119/64 17/15 51/40 -11 85/48 -7 255/128 -1 18/17
181 182 183 184 185 186 187 188 189 190
+11 +7 +26 +2 +6
-9 256/153 -17 30/17 -21 80/51 96/85 256/255 -36 28/17 128/119 25/21 75/56 -61 175/96

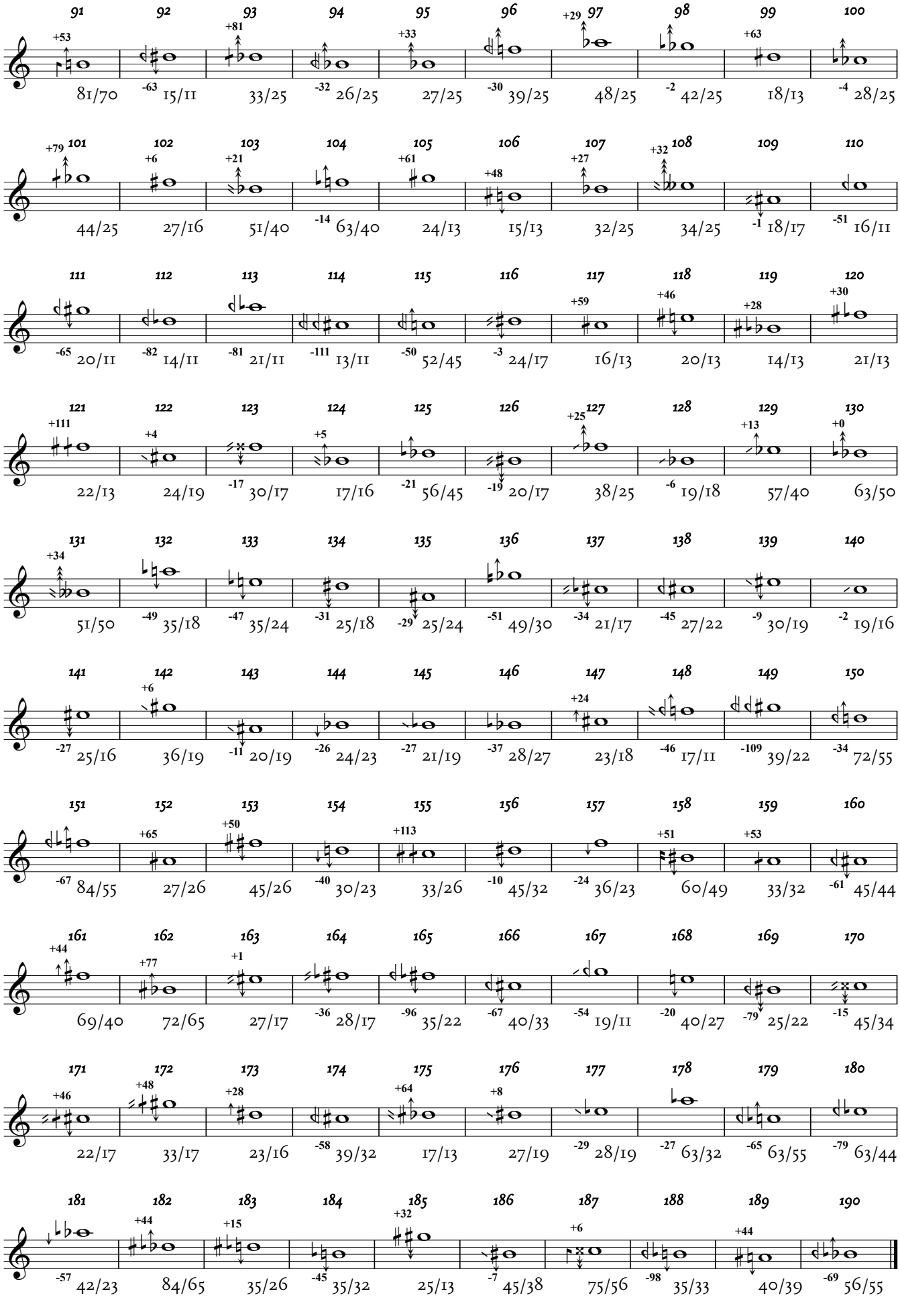
190 points in 23-limit harmonic space, 1/1 ≤ P < 2/1




91 92 93 94 95 96 97 98 99 100
+93 +75 +61 +73 +67 +20
69/49 69/35 69/56 -49 12/11 46/35 -17 7/5 33/20 -47 18/11 27/20 -46 13/10
101 102 103 104 105 106 107 108 109 110
+6 +63 +39
-59 13/8 -40 195/112 -24 117/112 -78 78/77 -44 39/20 -61 13/12 27/16 -16 21/20 18/13 225/196
111 112 113 114 115 116 117 118 119 120
+81 +79
-28 135/77 -14 108/77 -76 117/77 -46 65/63 180/91 120/91 -46 150/77 -91 195/154 -65 20/11 -16 144/77
121 122 123 124 125 126 127 128 129 130
+83 +50 +97
135/91 -10 45/32 -61 45/44 -45 27/22 -12 81/77 -44 225/154 45/26 108/91 -18 96/77 -34 80/77
131 132 133 134 135 136 137 138 139 140
+16 +14 +65 +14 +61 +24
-93 130/77 180/119 80/63 150/91 -17 30/17 120/119 -1 18/17 24/13 180/133 -9 30/19
141 142 143 144 145 146 147 148 149 150
+46 +130 +95 +146 +144 +18 +45 +93
20/13 -48 100/77 165/91 144/91 99/91 132/91 135/119 64/35 96/91 -0 100/63
151 152 153 154 155 156 157 158 159 160
+98 +67 +65 +128 +63 +1 +26
-7 180/161 225/182 27/26 162/91 110/91 100/91 150/119 -40 30/23 135/133 -15 45/34
161 162 163 164 165 166 167 168 169 170
+22 +15 +8 +8 +3 +66 +30 +31
85/56 95/56 225/224 150/133 -19 20/17 -3 24/17 225/119 165/119 144/119 36/25
171 172 173 174 175 176 177 178 179 180
+6 +22 +4
-7 45/38 36/19 240/133 -38 45/23 -27 25/16 -11 20/19 24/19 -51 16/11 -29 21/16 -81 21/11
181 182 183 184 185 186 187 188 189 190
+1 +50 +38 +12 +59 +62 +32
27/17 -4 16/9 144/133 16/15 68/35 16/13 -24 36/23 64/49 216/119 -13 102/77

190 points in 23-limit harmonic space, 1/1 ≤ P ≤ 2/1




91 92 93 94 95 96 97 98 99 100
+53 +81 +33 +29 +63
81/70 -63 15/11 33/25 -32 26/25 27/25 -30 39/25 48/25 -2 42/25 18/13 -4 28/25
101 102 103 104 105 106 107 108 109 110
+79 +6 +21 +61 +48 +27 +32
44/25 27/16 51/40 -14 63/40 24/13 15/13 32/25 34/25 -1 18/17 -51 16/11
111 112 113 114 115 116 117 118 119 120
+59 +46 +28 +30
-65 20/11 -82 14/11 -81 21/11 -111 13/11 -50 52/45 -3 24/17 16/13 20/13 14/13 21/13
121 122 123 124 125 126 127 128 129 130
+111 +4 +5 +25 +13 +0
22/13 24/19 -17 30/17 17/16 -21 56/45 -19 20/17 38/25 -6 19/18 57/40 63/50
131 132 133 134 135 136 137 138 139 140
+34
51/50 -49 35/18 -47 35/24 -31 25/18 -29 25/24 -51 49/30 -34 21/17 -45 27/22 -9 30/19 -2 19/16
141 142 143 144 145 146 147 148 149 150
+6 +24
-27 25/16 36/19 -11 20/19 -26 24/23 -27 21/19 -37 28/27 23/18 -46 17/11 -109 39/22 -34 72/55
151 152 153 154 155 156 157 158 159 160
+65 +50 +113 +51 +53
-67 84/55 27/26 45/26 -40 30/23 33/26 -10 45/32 -24 36/23 60/49 33/32 -61 45/44
161 162 163 164 165 166 167 168 169 170
+44 +77 +1
69/40 72/65 27/17 -36 28/17 -96 35/22 -67 40/33 -54 19/11 -20 40/27 -79 25/22 -15 45/34
171 172 173 174 175 176 177 178 179 180
+46 +48 +28 +64 +8
22/17 33/17 23/16 -58 39/32 17/13 27/19 -29 28/19 -27 63/32 -65 63/55 -79 63/44
181 182 183 184 185 186 187 188 189 190
+44 +15 +32 +6 +44
-57 42/23 84/65 35/26 -45 35/32 25/13 -7 45/38 75/56 -98 35/33 40/39 -69 56/55